2.12.

1) $f(x) := x^2 e^{-x^2}$, $D(f) = \mathbb{R}$

složená fce: e^y vnější

$-x^2$ vnitřní

pro $x \in \mathbb{R}$: $f'(x) = (x^2)' \cdot e^{-x^2} + x^2 (e^{-x^2})' = 2x e^{-x^2} + x^2 e^{-x^2}(-2x)$

$= \ldots$

Jinak: $f(x) = (g \circ h)(x)$, $g(y) = y \cdot e^{-y}$

$h(x) = x^2$

$$g'(y) = e^{-y} + y e^{-y}(-1) = e^{-y}(1-y)$$

$$h'(x) = 2x$$

$$f'(x) = \underbrace{e^{-x^2}(1-x^2)}_{g'(h(x))} \cdot \underbrace{2x}_{h'(x)}$$

4) <u>Špatný postup:</u>

$$f(x) = \lg(\lg(\sin x))$$

$$g(x) = \cancel{f'(x)} = \frac{1}{\lg(\sin x)} \cdot (\lg \sin x)' = \frac{1}{\lg \sin x} \cdot \frac{1}{\sin x} \cdot \cos x$$

$g(x)$ je definovaná na $(0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi) + 2k\pi$, $k \in \mathbb{Z}$

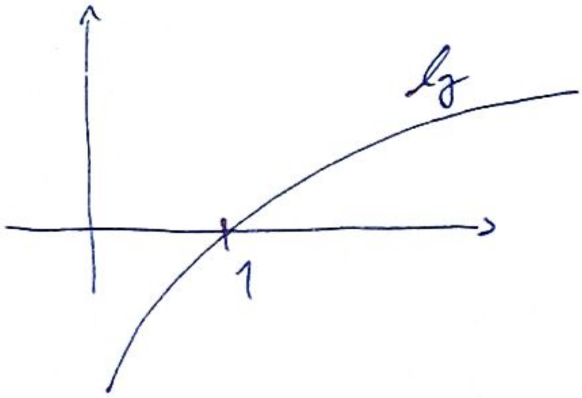
$$\mathbb{R} \setminus \left(\{k\pi;\ k \in \mathbb{Z}\} \cup \bigcup_{k \in \mathbb{Z}} [-\pi + 2k\pi, 0 + 2k\pi] \cup \left\{\frac{\pi}{2} + 2k\pi;\ k \in \mathbb{Z}\right\} \right)$$

Je $g = f'$ na $D(g)$?

$D(f) = \emptyset$

($\sin x > 0$ ..... $x \in (0, \pi) + 2k\pi$, $k \in \mathbb{Z}$,

$\lg \sin x > 0$, $\sin x > 1 \Rightarrow x \in \emptyset$)



$$\lim_{x\to 0} \frac{\lg\left(e^x\left(\frac{x^2}{e^x}+1\right)\right)}{\lg\left(e^{2x}\left(\frac{x^4}{e^{2x}}+1\right)\right)} = \lim_{x\to 0} \frac{x + \lg\left(1+x^2e^{-x}\right)}{2x + \underbrace{\lg\left(1+x^4e^{-2x}\right)}_{\to 0}} =$$

$$= \lim_{x\to 0} \frac{1 + \frac{\lg\left(1+x^2e^{-x}\right)}{x^2e^{-x}} \cdot x e^{-x}}{2 + \underbrace{\frac{\lg\left(1+x^4e^{-2x}\right)}{x^4e^{-2x}}}_{\to 1} \cdot \underbrace{x^3e^{-2x}}_{\to 0}} = \frac{1}{2}$$

$$\frac{\lg e^x + \lg\left(1+x^2e^{-x}\right)}{} = \lg\left(\frac{e^x-1+1}{e^x-1}\right)\left(\frac{e^x-1}{x}\right)\cancel{x}$$

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2) $f(x) = \lg\left(\frac{x^2-1}{x^2+1}\right)$

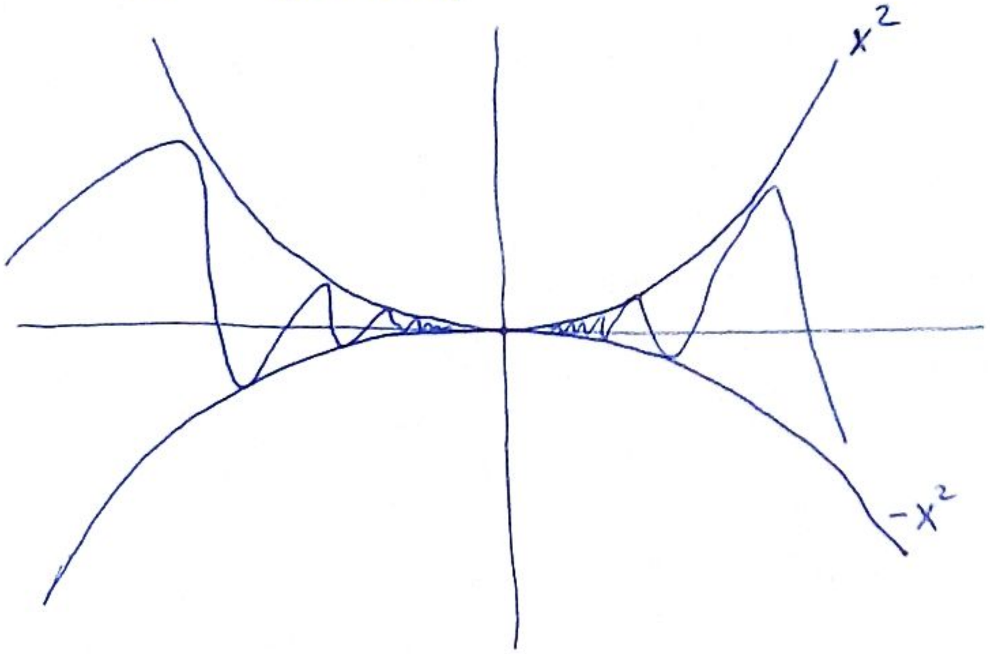
$D(f)$: $\frac{x^2-1}{x^2+1} > 0$; $x \in (-\infty,-1) \cup (1,+\infty)$

$x \in D(f)$: $$f'(x) = \frac{1}{\frac{x^2-1}{\cancel{x^2+1}}} \cdot \frac{2x(\cancel{x^2}+1) - 2x(\cancel{x^2}-1)}{(x^2+1)^{\cancel{2}}} =$$

$$= \frac{4x}{x^4-1}$$

3) $f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{x^2 \sin\left(\frac{1}{\sqrt[3]{x}}\right) \cancel{-0}}{x \cancel{-0}}$

$= \lim_{x\to 0} \underbrace{x}_{\to 0} \underbrace{\sin\left(\frac{1}{\sqrt[3]{x}}\right)}_{\text{omezená}} = 0$



5) $f(x) := x^{\lg x} = \exp(\lg x \cdot \lg x) = \exp((\lg x)^2)$

$\mathcal{D}(f) = \{x > 0\} = (0, +\infty)$

$x \in \mathcal{D}(f): \; f'(x) = \exp((\lg x)^2) \cdot 2 \lg x \cdot \frac{1}{x}$

Př: $\operatorname{arctg} x + \operatorname{arccotg} = C$

3) $f(x) = x^2 \sin\left(\frac{1}{\sqrt[3]{x}}\right)$

$x \neq 0: \quad f'(x) = 2x \sin\left(\frac{1}{\sqrt[3]{x}}\right) + x^2 \cos\left(\frac{1}{\sqrt[3]{x}}\right)\left(-\frac{1}{3}\right) x^{-\frac{4}{3}}$

$$\left(\frac{1}{\sqrt[3]{x}}\right)' = \left(x^{-1/3}\right)' = -\frac{1}{3}\, x^{-\frac{1}{3}-1}$$

$$= \frac{0 \ldots \; - 1 \cdot \frac{1}{3} x^{-2/3}}{\left(\sqrt[3]{x}\right)^2} =$$