Spočtěte $\int_{\Omega} \sin \sqrt{x^2+y^2} \, d\lambda^2(x,y)$ kde

$\Omega = \{ (x,y) \in \mathbb{R}^2 ; \pi^2 \leq x^2 + y^2 \leq 4\pi^2 \}$

Ω je uz. $\Rightarrow$ Ω je měřitelná

Funkce je spojitá na $\mathbb{R}^2$ $\Rightarrow$ je měřitelná

Použijeme polární souřadnice.

(*) $\psi : (0,\infty) \times (-\pi, \pi) \longrightarrow \mathbb{R}^2$

$$\left.\begin{array}{l} x = r \cos \alpha \\ y = r \sin \alpha \end{array}\right\} \Rightarrow x^2 + y^2 = r^2$$

$$\psi^{-1}(A) = \{ (r,\alpha) \in (0,\infty) \times (-\pi, \pi) ; \ r \in [\pi, 2\pi] \}$$

$$|J\psi| = r$$

$$\int_M \sin\sqrt{x^2+y^2}\, d\lambda^2(x,y) \overset{(*)}{=} \int_{\varphi^{-1}(M)} r\cdot \sin r \, d\lambda^2(r,\alpha)$$

Fubini
integrand je ≤ 0

$$\overset{=}{=} \int_{\pi}^{2\pi}\int_{-\pi}^{\pi} r \sin r \, d\alpha\, dr = 2\pi \int_{\pi}^{2\pi} r \sin r \, dr$$

per partes

$$= 2\pi \left(\left[-r\cos r \right]_{\pi}^{2\pi} - \int_{\pi}^{2\pi} -\cos r \, dr \right) =$$

$$= 2\pi \left(-2\pi - \pi + \left[\sin r \right]_{\pi}^{2\pi} \right) = -6\pi^2.$$