PSt – 3. cvič.

① X_i .... bodový zisk v i-té otázce, $X_1,\dots,X_{20}$ n.n.v. $\sim Bern(p)$, $X = \sum_{i=1}^{20} X_i$

a) $EX_i = P(\text{zná odpověď}) = p$

$\sum x P(X_i = x) = 0 \cdot P(\text{nezná}) + 1 \cdot P(\text{zná})$

$EX = 20p$

b) $EX_i =$

$E(X_i \mid \text{zná}) P(\text{zná}) + E(X_i \mid \text{nezná}) \cdot P(\text{nezná})$

$1 \cdot p + (1-p) \cdot E(X_i \mid \text{nezná})$

$E(X_i \mid \text{tipne dobře}) P(\text{tipne}) + E(X_i \mid \text{tipne šp.}) P(\text{špatně})$

$= \frac{1}{4} \cdot 1 + \frac{3}{4} \cdot \frac{-1}{4} = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16} > 0$

$= p + (1-p)\frac{1}{16}$

$EX = 20\left(p + (1-p)\frac{1}{16}\right)$

c) $-\frac{1}{3}$

$Y = n-1$

② X = odměna $\quad P(X = 2^n) = \frac{1}{2^n} \quad n \sim Geom\left(\frac{1}{2}\right)$

$EX = \sum_{x \in \mathrm{Im} X} x P(X=x) = \sum_{n=1}^{\infty} 2^n \frac{1}{2^n} = \sum_{n=1}^{\infty} 1 = \infty$

$P(X > 100) = 2^{-100}$

průměr z n 2

průměr z X .... ∞

③ a) $Y = X^2 \quad Y \geq 0 \quad EY = 0 \Rightarrow P(Y \neq 0) = 0$

$Y = 0$ s.j.

b) plyne z a)

$\operatorname{var} X = E\underbrace{(X-\mu)^2}_{} = 0 \Rightarrow Y = 0$ s.j.

$X = \mu$ s.j.

$\mu = EX \quad Y \geq 0$

(4) $EX = \sum_{n=0}^{\infty} P(X>n)$ $X:\Omega \to \mathbb{N}_0 = \{0,1,2,\dots\}$

$$EX = \sum_{k=0}^{\infty} k\,P(X=k) = \sum_{k=0}^{\infty}\sum_{n=0}^{k-1} \underbrace{P(X=k)}_{\geq 0}$$

$$= \sum_{n=0}^{\infty}\sum_{k=n+1}^{\infty} P(X=k) = P(X>n)$$

$\boxed{\text{Satz: } 7, 8}$

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(5) a) $\operatorname{var} X = \mathbb{E}(X^2) - (\mathbb{E}X)^2$

$$\operatorname{var}(aX) = E(aX)^2 - (\mathbb{E}(aX))^2 = E(a^2X^2) - (a\mathbb{E}X)^2$$

$\omega \mapsto aX(\omega)$

$$= a^2\left(\mathbb{E}X^2 - (\mathbb{E}X)^2\right) = a^2 \cdot \operatorname{var}(X)$$

$\boxed{E(aX) = a\cdot EX}$

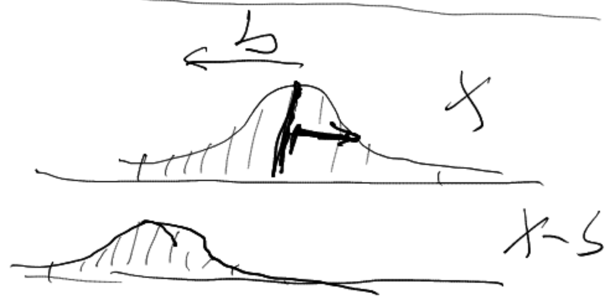
$\boxed{E(a^2 \cdot X^2) = a^2 \cdot E(X^2)}$

b)

$\boxed{\operatorname{var}(X-b) = \operatorname{var}(X)}$

$$\mathbb{E}\left((X-\mathbb{E}X)^2\right)$$

$$E\left[(X-b) - \mathbb{E}(X-b)\right]^2$$

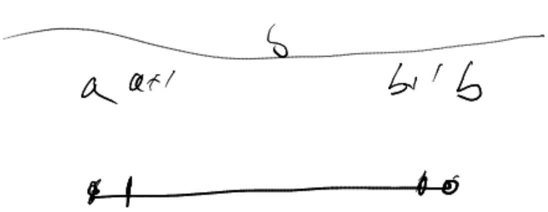


(7) $X = a, a+1, a+2, \dots$ je s. pst: $\frac{1}{b-a+1}$

$$\mathbb{E}X = \sum_{x \in \operatorname{Im} X} x \cdot P(X=x)$$

$$= \frac{a + (a+1) + \dots + b}{b-a+1}$$

$$= \frac{a+b}{2}$$



bůno $a=1$. (jinak: $X \rightsquigarrow X-(a-1)$

$1, \dots, b-a+1$)

b

$$\mathbb{E}X^2 = \sum x^2 P(X=x) = \frac{1^2+2^2+\dots+b^2}{b} = \frac{\frac{1}{3} b\,(b+\frac{1}{2})(b+1)}{b}$$

$1 \dots b$

$$= \frac{(b+\frac{1}{2})(b+1)}{3} \sim \frac{b^2}{3}$$

$\sigma = \sqrt{\operatorname{var} X}$

(8) a) ✓

b)

(9) $X = X_1 + \dots + X_n$ $\quad \mathbb{E}X_i = p$ $\quad \operatorname{var} X_i = p(1-p)$

$\mathbb{E}X = \mathbb{E}X_1 + \dots + \mathbb{E}X_n = np$

$\operatorname{var} X = \operatorname{var} X_1 + \dots + \operatorname{var} X_n = np(1-p)$

jen pro n.n.v.

$$\Omega \xrightarrow{X} \mathbb{R}$$

$\omega \longmapsto X(\omega)$ (sax)

$\longmapsto (X(\omega))^2 \dots$ n.v. X^2 (sax)^2