5. cvič.

(3) $T \sim Exp(\lambda)$ $\quad f_T(t) = \lambda e^{-\lambda t}$

$F_T(t) = 1 - e^{-\lambda t}$ $\quad f_T = F_T'$

a) $\mathbb{E}T = \frac{1}{\lambda} = 4$ $\quad \lambda = \frac{1}{4}$

$F_T(t) = 1 - e^{-\frac{t}{4}}$

b) $P(T > 4) = 1 - P(T \le 4) = 1 - F_T(4) = e^{-\frac{4}{4}} = \frac{1}{e} = 0.368$

jaký je medián $T$? $\quad F_T(m) = \frac{1}{2} = 1 - e^{-\frac{t}{4}}$

$e^{-\frac{t}{4}} = \frac{1}{2}$ $\quad \frac{t}{4} = \ln 2$

$t = 4 \ln 2 = 2.8$ ~~2.8~~ 2.77

c) $P(3 \le T \le 5) = F_T(5) - F_T(3)$

$= e^{-\frac{3}{4}} - e^{-\frac{5}{4}} = 0.186$

(11) $X \ge 0$

$\mathbb{E}X = \int_0^\infty P(X > t)\, dt$ $\quad \sum_t P(X > t)$

$\int_0^\infty x f(x) = [x(F-1)]_0^\infty - \int_0^\infty 1 \cdot (F-1)$

u' v

$\infty \cdot 0$ ?

$\int_0^\infty \underbrace{1 - F(t)}_{P(X > t)}\, dt$

(9) a) $P(|Z| \le 1) = P(Z \le 1) - P(Z \le -1)$ $= P(Z \ge -1)$

$= \Phi(1) - \Phi(-1)$

$= 0.84 - 0.16 = 0.68$

b)

c)


10 b) $X \sim N(40, 10^2)$

$P(50 \le X \le 70)$

$P(10 \le X - 40 \le 30)$

$P(1 \le \frac{X-\mu}{\sigma} \le 3) = \Phi(3) - \Phi(1)$

$\frac{X-\mu}{\sigma} = Z \sim N(0,1)$

d) $X \sim N(\mu, \sigma^2)$ $\quad Z = \frac{X-\mu}{\sigma} \sim N(0,1)$

$|Z| \le 1 \iff \left|\frac{X-\mu}{\sigma}\right| \le 1$

$\iff |X - \mu| \le 1 \cdot \sigma$