Seštete $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{2^n n!}$

Položme $\displaystyle f(x) = \sum_{n=1}^{\infty} x^n \frac{n^2}{n!}$. Rad

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^2}{n^2} \cdot \frac{1}{n+1} = 0 \quad \text{i.e.}$$

rada konvergaje na $\mathbb{R}$.

Platí: $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$x e^x = \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!}(n+1) = \sum_{n=1}^{\infty} \frac{x^n}{n!} n$$

$$x^2 e^x = \sum_{n=2}^{\infty} \frac{x^n n(n-1)}{n!} = \sum_{n=1}^{\infty} \frac{x^n n(n-1)}{n!}$$

vše na $\mathbb{R}$. (Poloměry konvergence stejné jak. u e.)

$$f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n!} n(n-1) + \sum_{n=1}^{\infty} \frac{x^n}{n!} n =$$

$$= x e^x + x^2 e^x .$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n \, n^2}{n!} = f\left(-\frac{1}{2}\right) = -\frac{1}{4} e^{-\frac{1}{2}} .$$