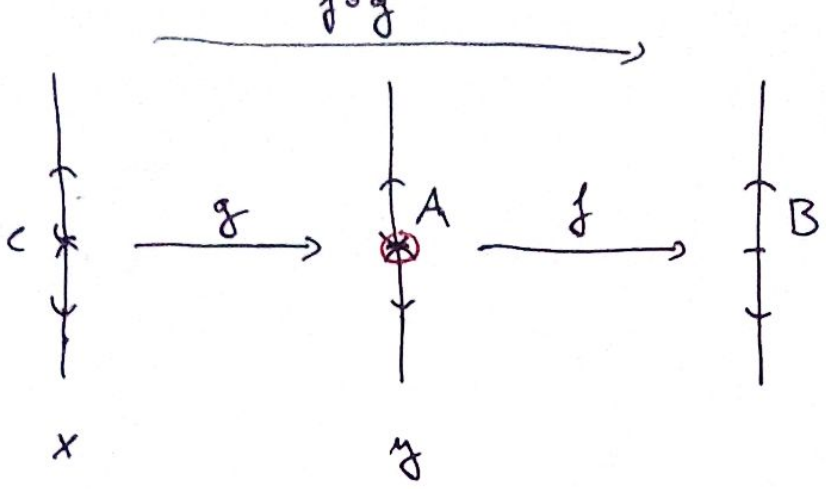
Cvičení mma 101 – 11.11.

V3.6:




Příklad: $g(x) = A \ \forall x$ ; $f(y) = \begin{cases} B & y \neq A \\ \text{nedef.} & y = A \end{cases}$

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- $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{\sin^2\frac{x}{2} + \cancel{\cos^2\frac{x}{2}} - \left(\cancel{\cos^2\frac{x}{2}} - \sin^2\frac{x}{2}\right)}{x^2}$

Součtové vzorce pro sin a cos.

$\cos(x-x) = 1 = \sin^2 x + \cos^2 x$

$\cos(2x) = \cos(x+x) = \cos^2 x - \sin^2 x$

$= \lim_{x\to 0} \frac{\sin^2\frac{x}{2}}{\frac{x^2}{2}}$

$= \lim_{x\to 0} \underbrace{\frac{\sin\frac{x}{2}}{\frac{x}{2}}}_{\to 1} \cdot \underbrace{\frac{\sin\frac{x}{2}}{\frac{x}{2}}}_{\to 1} \cdot \frac{1}{2} \overset{AL}{=} \frac{1}{2}$

Věta 3.6: $\lim \frac{\sin\frac{x}{2}}{\frac{x}{2}}$

$g(x) := \frac{x}{2}$

$f(y) := \frac{\sin y}{y}$

$\Rightarrow (f\circ g)(x) = \frac{\sin\frac{x}{2}}{\frac{x}{2}}$

$c := 0$ ; $A := \lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{x}{2} = 0$ ; $B := \lim_{y\to A} f(y) = 1$

(P) maj. $P(0)$ & $g \neq 0$ ✓

- $g$ je prostá
- $g(x) = 0 \Leftrightarrow x = 0$

$\Rightarrow \lim_{x\to 0} \frac{\sin\frac{x}{2}}{\frac{x}{2}} = 1$

$$\lim_{x\to 0}\frac{\ln(1+x)}{x}=\lim_{x\to 0}\frac{\ln(1+x)}{1+x-1}$$

$$=\lim_{x\to 0}\frac{\ln(1+x)}{\exp(\ln(1+x))-1}=\lim_{x\to 0}\frac{1}{\frac{\exp(\ln(1+x))-1}{\ln(1+x)}}$$

$$f(y):=\frac{e^y-1}{y}\qquad \lim_{y\to 0} f(y)\overset{AL}{=}1$$

$$g(x):=\ln(1+x)\quad \text{je pravá}' \to (P)$$

$$\lim_{x\to 0} g(x)\overset{\uparrow}{=}\ln(1)=0 \Rightarrow \equiv 1$$

ln jinj. fce  VS. 6

1) $x^{100}-2x+1=x^{100}-x^2+x^2-2x+1$

$$=x^2(x^{98}-1)+(x-1)^2$$

$$\uparrow$$

$$*^{98}-1=(x-1)(x^{97}+x^{96}+\ldots+x+1)$$

5) $$\lim_{x\to 0}\frac{x^2}{\sqrt{1+x\sin x}-\sqrt{\cos x}}=\lim_{x\to 0}\frac{x^2\left(\sqrt{1+x\sin x}+\sqrt{\cos x}\right)}{1-\cos x+x\sin x}$$

$$\overset{AL}{=}2\cdot\lim_{x\to 0}\frac{1}{\frac{1-\cos x}{x^2}+\frac{x\sin x}{x\cdot x}}=2\cdot\frac{1}{\frac{1}{2}+1}$$

$$=\frac{4}{3}$$

6) $A^3+B^3=(A+B)(A^2-AB+B^2)$

$-4+6-1 = \underline{1}$

$$\lim_{x\to 2} \frac{1}{(x-2)^2} \cdot \underbrace{\frac{-x^2+3x-1}{x(x-1)}}_{\xrightarrow[x\to 2]{} \frac{1}{2}} = +\infty$$

$\xrightarrow[x\to 2]{} +\infty$

⇑ (9)

$g(x) = (x-2)^2$

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$$\tilde{x} - 1 \leq [\tilde{x}] \leq \tilde{x} \quad ; \quad \tilde{x} \in \mathbb{R}$$

$$x\left(\frac{1}{x} - 1\right) \leq \underbrace{x\left[\frac{1}{x}\right]}_{\xrightarrow[x\to 0]{} 1} \leq x \cdot \frac{1}{x} = 1$$

$=$

$1 - x$

$\downarrow x \to 0$

$1$

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