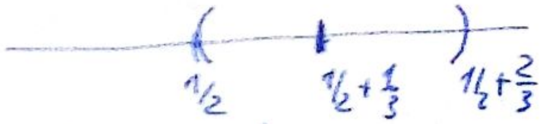
## Cvičení nmma 101 – 10.11.20

## Nápověda k 11/8c

$\{a_n\}$ daná, $\mathcal{H}(\{a_n\}) = [0,1]\setminus\{\frac{1}{2}\}$

Krok 3: $\exists \{n_k^3\}_{k=1}^{+\infty} : \{a_{n_k^3}\}_{k=1}^{+\infty}$ je vybraná z $\{a_n\}$

$$a_{n_k^3} \longrightarrow \frac{1}{2} + \frac{1}{3}$$




$\Rightarrow$ $\cancel{\exists n^{3,0} \in \mathbb{N}, \forall k > n^{3,0}}$

$\exists k_0^3 \in \mathbb{N} : \forall k > k_0^3 : a_{n_k^3} \in \left(\frac{1}{2}, \frac{1}{2}+\frac{2}{3}\right)$.

Vol $n_3 = n^3_{k_0^3+1}$, pak $a_{n_1} \in \left(\frac{1}{2}, \frac{1}{2}+\frac{2}{3}\right)$

Krok 4: $\exists \{n_k^4\}_{k=1}^{+\infty}$; $a_{n_k^4} \longrightarrow \frac{1}{2}+\frac{1}{4}$, $k \to +\infty$

$\exists k_0^4 \in \mathbb{N} : \forall k > k_0^4 : a_{n_k^4} \in \left(\frac{1}{2}, \frac{2}{4}\right)$

Vol $n_4 = n_k^4$ tak, aby $k > k_0^4$ a $n_4 > n_3$.

Krok j: Nahraď v Kroku 4, $4 \to j$ a požaduj $n_j > n_{j-1}$.

Pak $\{a_{n_j}\}$ je vybraná z $\{a_n\}_{n=1}^{+\infty}$ a

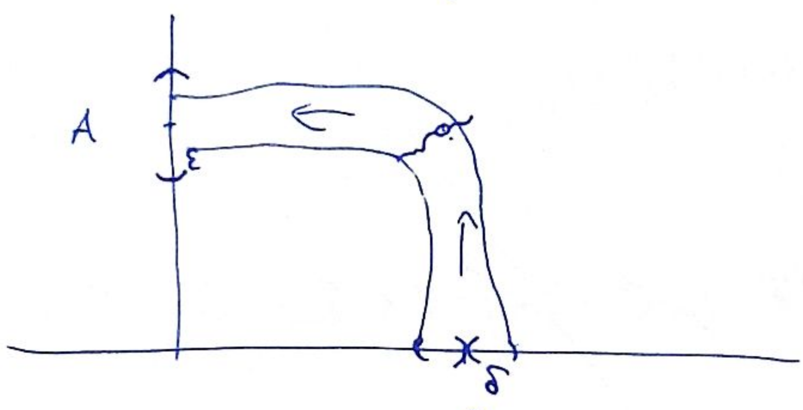
$$\frac{1}{j} < a_{n_j} - \frac{1}{2} < \frac{1}{j} \Rightarrow a_{n_j} \xrightarrow{j\to+\infty} \frac{1}{2}$$

$\perp$.

## Limity funkcí

$f: P(a) \to \mathbb{R}$ má $\lim_{x\to a} f(x) = A \in \mathbb{R}^*$; $a \in \mathbb{R}^*$

$\overset{df}{\Longleftrightarrow}$ $\forall \varepsilon > 0, \exists \delta > 0: f(P(a,\delta)) \subset \mathcal{U}(A,\varepsilon)$

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$\lim_{x\to 1} x = 1$ z definice: $a = 1, A = 1$; $f(x) = x$

Fix $\varepsilon > 0$, hledám $\delta > 0$: $\forall x \in P(1,\delta) = (1-\delta, 1+\delta)\setminus\{1\}$

$$|f(x) - 1| < \varepsilon.$$

$$|x - 1| < \varepsilon \Rightarrow x \in (1-\varepsilon, 1+\varepsilon)$$

$\Rightarrow$ stačí volit $\delta := \varepsilon$.

Podobně: $\lim_{x\to a} x = a$.

AL: Je-li $P: \mathbb{R} \to \mathbb{R}$ polynom ($P(x) = \sum_{k=0}^{n} a_k x^k$; $n \in \mathbb{N}$, $a_k \in \mathbb{R}$)

pak $\lim_{x\to a} P(x) = P(a)$ pro $a \in \mathbb{R}$.

12/1 $\infty$

$$\lim_{x\to 1} \frac{\overbrace{x^2+2x+3}^{\xrightarrow{x\to 1} 6}}{\underbrace{x^2+1}_{\xrightarrow{x\to 1} 2}} \overset{AL}{=} \frac{6}{2} = 3$$

$$\lim_{x\to 1} \frac{\overbrace{x^2+2x-3}^{\xrightarrow{x\to 1} 0}}{\underbrace{x^2-1}_{\xrightarrow{x\to 1} 0}} \overset{AL}{=} \cancel{\frac{0}{0}} \quad \text{nelze}$$

Hledáme 0, tj. $x-1$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}\overbrace{(x+3)}^{\to 4}}{\cancel{(x-1)}\underbrace{(x+1)}_{\to 2}} \overset{AL}{=} 2$$

Ke 3: $x^3 - 12x + 16 : x - 2 = x^2 + 2x - 8$

$-(x^3 - 2x^2)$

$2x^2 - 12x$

$-(2x^2 - 4x)$

$-8x + 16$

$-(-8x + 16)$

$0$

K6: $\lim_{x\to 0} \sqrt{x+1} = 1$

12/6: $$\lim_{x\to 0} \frac{\overbrace{\sqrt{x+1}-\sqrt{1-x}}^{\to 0}}{\underbrace{\sqrt[3]{x+1}-\sqrt[3]{1-x}}_{\to 0}} =$$

$$\lim_{x\to 0} \frac{\overbrace{x+1-(1-x)}^{2\cancel{x}}}{\sqrt{x+1}+\sqrt{-x+1}} \cdot \frac{\left(\sqrt[3]{x+1}\right)^2+\sqrt[3]{x+1}\,\sqrt[3]{1-x}+\left(\sqrt[3]{1-x}\right)^2}{\underbrace{x+1-(1-x)}_{2\cancel{x}}}$$

$$\overset{AL}{=} \frac{2}{2}\cdot\frac{3}{2} = \frac{3}{2}$$

$$A^n-B^n=(A-B)(A^{n-1}+A^{n-2}B+\dots+B^{n-1})$$

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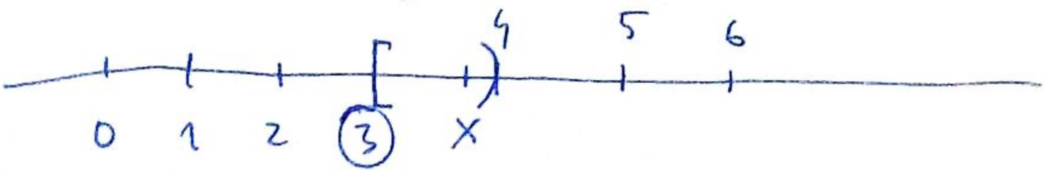
$$\lim_{x-1} \frac{x+x^2+\dots+x^n-n}{x-1} =$$

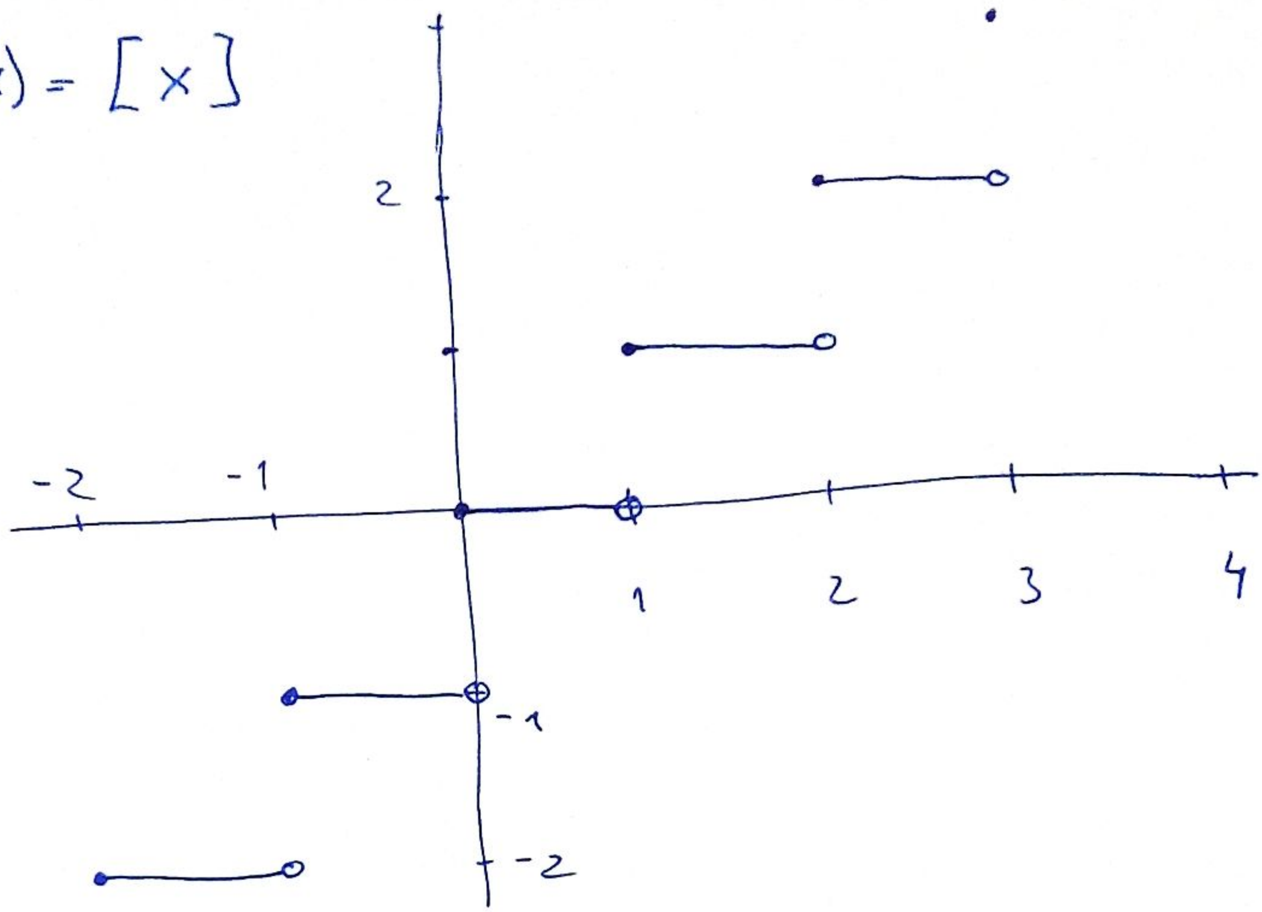
$$\lim_{x\to 1} \frac{x-1+x^2-1+\dots+x^n-1}{x-1} =$$

$$\lim_{x\to 1} \sum_{k=1}^{n} \frac{x^k-1}{x-1} \overset{AL}{=} \sum_{k=1}^{n} \lim_{x\to 1} \frac{x^k-1}{x-1}$$

12/8 $[x] = k$, kde $x \in [k, k+1)$; $k \in \mathbb{Z}$



$f(x) = [x]$




$$\lim_{x \to +\infty} \frac{x}{[x]} \; ; \qquad x - 1 \leq [x] \leq x$$

$$1 \leq \frac{x}{[x]} \leq \frac{x}{x-1} \xrightarrow{x \to +\infty} 1$$

V3.5 iii) $\Rightarrow \lim_{x \to +\infty} \frac{x}{[x]} = 1$