Konv. kódovač $K: \mathbb{F}^b((D)) \longrightarrow \mathbb{F}^c((D))$ $\qquad uG(D) = v$

- závislý na zvolenej báze

- lineárny $\alpha, \beta \in \mathbb{F}$, $u, v \in \mathbb{F}^b((D))$: $K(\alpha u + \beta v) = \alpha K(u) + \beta K(v)$
- časovo invariantný: $K(uD) = K(u)D$
- konečný počet stavov
- prechodová f-cia: $S \times \mathbb{F}^b \longrightarrow S \times \mathbb{F}^c$ $\qquad S$ dimenzie s; $s_0 = 0$

$$(s_i, u_i)\ \begin{array}{c|cc} & s & c \\ \hline s & & \\ b & & \end{array}\begin{pmatrix} \mathcal{A} & \vdots & \mathcal{C} \\ \cdots & & \cdots \\ \mathcal{B} & \vdots & \mathcal{D} \end{pmatrix} = (s_{i+1}, v_i)$$

$s_1 + s_2 D + s_3 D^2$

$\sum s_i = s$

$s_0 + s_1 D + s_2 D^2 \dots$

$\sum_{i=0}^{\infty} s_{i+1} = s \cdot D^{-1}$

$sD^{-1} = s\mathcal{A} + u\mathcal{B}$

$$s_{i+1} = s_i \mathcal{A} + u_i \mathcal{B}$$
$$v_i = s_i \mathcal{C} + u_i \mathcal{D} \longrightarrow v = s \cdot \mathcal{C} + u \cdot \mathcal{D}$$

$$v = u\big(\underbrace{\mathcal{B}(I_s D^{-1} - \mathcal{A})^{-1}\mathcal{C}}_{G(D)} + \mathcal{D}\big)$$

$s(I_s D^{-1} - \mathcal{A}) = u\mathcal{B}$

$s = u\mathcal{B}(I_s D^{-1} - \mathcal{A})^{-1}$

Príklad 1. v $\mathbb{F}_2$

Nájdite generujúcu maticu G pre kódovač zadaný

$$\mathcal{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \quad \mathcal{C} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \mathcal{D} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix},$$

kde $s_{i+1} = s_i \mathcal{A} + u_i \mathcal{B}, \dots$

$$G = \mathcal{B}(\underline{I_3 D^{-1} - \mathcal{A}})^{-1}\mathcal{C} + \mathcal{D} = (\mathcal{B}(I_3 - D\mathcal{A})^{-1}\mathcal{C})D + \mathcal{D}$$

$$(D^{-1}(I_3 - D\mathcal{A}))^{-1} = (I_3 - D\mathcal{A})^{-1}D \quad \text{nad } \mathbb{F}_2(D)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} D & 0 & D \\ 0 & D & 0 \\ D & D & 0 \end{pmatrix} = \begin{pmatrix} 1+D & 0 & D \\ 0 & 1+D & 0 \\ D & D & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1+D & 0 & D & 1 & 0 & 0 \\ 0 & 1+D & 0 & 0 & 1 & 0 \\ D & D & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & D & 1+D & 1 & 0 & 1 \\ 0 & 1+D & 0 & 0 & 1 & 0 \\ D & D & 1 & 0 & 0 & 1 \end{array}\right) \sim$$

$\cancel{D^3} + \cancel{D} + \cancel{D^2} + D^3$
$+ \cancel{D} + \cancel{D^2}$

$$\begin{pmatrix} \frac{1}{1+D+D^2} & \frac{D^2}{1+D^3} & \frac{D}{1+D+D^2} \\ 0 & \frac{1}{1+D} & 0 \\ \frac{D}{1+D+D^2} & \frac{D}{1+D+D^2} & \frac{1+D}{1+D+D^2} \end{pmatrix} = \frac{1}{1+D^3}\begin{pmatrix} 1+D & \cancel{D+D^2+D^3}\ D^2 & D+D^2 \\ 0 & 1+D+D^2 & 0 \\ D+D^2 & D+D^2 & 1+D^2 \end{pmatrix}$$

$$(\ \ )^{-1} = \frac{1}{1+D^3}\begin{pmatrix} 1+D & \cancel{D+D^2+D^3}\ \ D^2 & D+D^2 \\ 0 & 1+D+D^2 & 0 \\ D+D^2 & D+D^2 & 1+D^2 \end{pmatrix}$$

$$G = \mathcal{B}(\ \ )^{-1}\mathcal{C}\cdot D + \mathcal{D}$$

$$\frac{1}{1+D^3}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \cdot \begin{array}{c} 1 \\ 1 \\ 0 \end{array}\begin{pmatrix} 1+D & D^2 & D+D^2 \\ 0 & 1+D+D^2 & 0 \\ D+D^2 & D+D^2 & 1+D^2 \end{pmatrix} = \frac{1}{1+D^3}\begin{pmatrix} 1+D & 1+D & D+D^2 \\ D+D^2 & 1 & 1+D^2 \end{pmatrix}$$

$$\frac{1}{1+D^3}\begin{pmatrix} 1+D & 1+D & D+D^2 \\ D+D^2 & 1 & 1+D^2 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \frac{1}{1+D^3}\begin{pmatrix} D+D^2 & 0 & 1+D \\ 1+D^2 & 1+D+D^2 & D+D^2 \end{pmatrix}$$

$$\frac{1}{1+D^3}\begin{pmatrix} D^2+D^3 & 0 & D+D^2 \\ D+D^3 & D+D^2+D^3 & D^2+D^3 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} =$$

$$\frac{1}{1+D^3}\begin{pmatrix} D^2+D^3 & 0 & 1+D+D^2+D^3 \\ 1+D & D+D^2+D^3 & 1+D^2 \end{pmatrix}$$

Príklad 2

Nájdite $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ pre kódovač zadaný! $k=2$, $c=3$

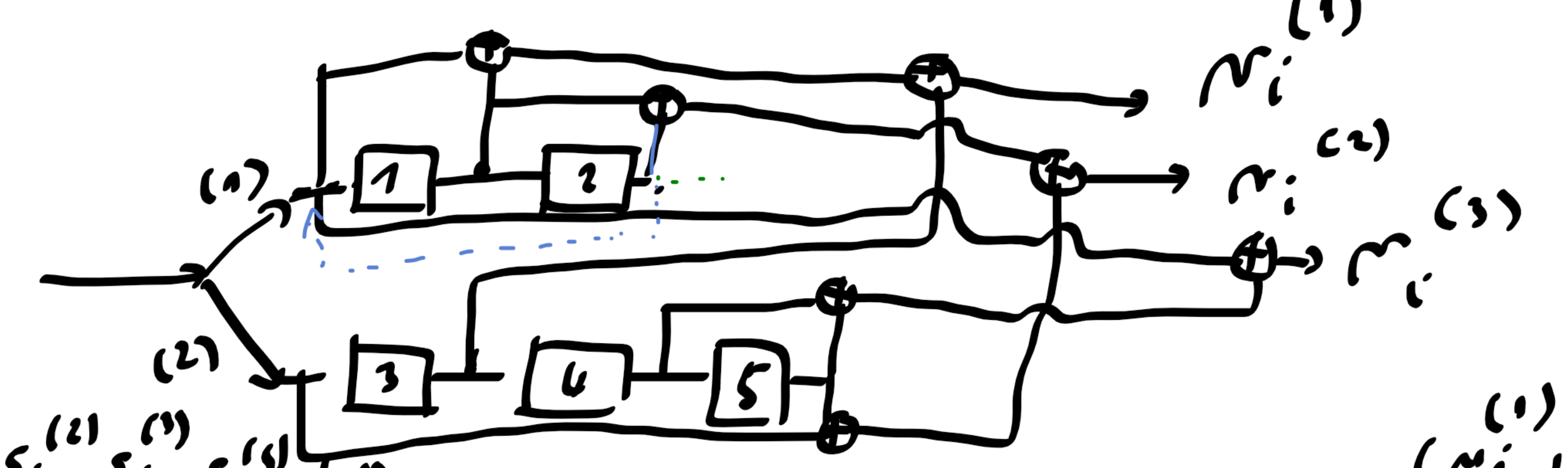
$$G = \begin{array}{c} \vec{m}^{(1)} \\ \vec{m}^{(2)} \end{array} \begin{pmatrix} 1+D & D+D^2 & 1 \\ D & 1+D^3 & D^2+D^3 \end{pmatrix}$$

$n_i = (n_i^{(1)}, n_i^{(2)}, n_i^{(3)})$

$m_i = (m_i^{(1)}, m_i^{(2)})$

$$s_{i+1} = s_i \mathcal{A} + m_i \mathcal{B}$$

$$n_i = s_i \mathcal{C} + m_i \mathcal{D}$$



$(s_i^{(1)}, s_i^{(2)}, s_i^{(3)}, s_i^{(4)}, s_i^{(5)})$

$$\mathcal{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$(m_i^{(1)}, m_i^{(2)})$

$$\mathcal{B} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\mathcal{C} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\mathcal{D} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$