1)

$$f(x,y) = (x^2 + 7y^2)\, e^{-(2x^2+y^2)}, \quad M = \{x^2+4y^2 \leq 1\}$$

Naleznete globální extrémy f na M.

- M je uz.
- M je omez. neboť $M \subseteq B(0,1)$

$$\left[x^2 + y^2 \leq x^2 + 4y^2 \right]$$

$\Rightarrow$ M je kompakt $\Rightarrow$ f nabývá maxima,

$$M = \underbrace{\{x^2 + 4y^2 < 1\}}_{M_1} \cup \underbrace{\{x^2 + 4y^2 = 1\}}_{M_2}$$

$$\frac{\partial f}{\partial x}(x,y) = 2x\, e^{-(2x^2+y^2)} + (x^2 + 7y^2)\, e^{-(2x^2+y^2)}(-4x)$$

$$\frac{\partial f}{\partial y}(x,y) = 14y\, e^{-(2x^2+y^2)} + (x^2 + 7y^2)\, e^{-(2x^2+y^2)}(-2y)$$

$$\nabla f(x,y) = e^{-(2x^2+y^2)}\, 2\left(x\left(1 - 2x^2 - 14y^2\right), y\left(7 - x^2 - 7y^2\right)\right)$$

Hledáme body M_1 kde $\nabla f = 0$.

$$\nabla f(x,y) = 0 \iff x = y = 0 \; \ldots \; (0,0) \in \Lambda_1$$

$$\lor$$

$$x = 0, \; y^2 = 1 \; \ldots \; (0, \pm 1) \notin \Lambda_1$$

$$\lor$$

$$y = 0, \; x^2 = \frac{1}{2} \; \ldots \; \left(\pm \frac{1}{\sqrt{2}}, 0\right) \in \Lambda_1$$

Vyšetříme M_2 pomocí Věty o mult.

$$g(x,y) = x^2 + 4y^2 \in C^\infty(\mathbb{R}^2)$$

$\nabla g(x,y) = (2x, 8y)$ je $0 \Leftrightarrow (x,y) = (0,0) \notin M_2$

i.e. i) není na M_2 splněná.

Z pod. ii) dostávám rovnice

$$x^2 + 4y^2 = 1$$
$$x - 2x^3 - 14xy^3 = \lambda 2x$$
$$7y - 7y^3 - yx^2 = \lambda 8y$$

a) $x = 0$. Z první rovnice $y^2 = \frac{1}{4}$, λ ex.
Dostávám body $(0, \pm \frac{1}{2})$.

b) $y = 0 \Rightarrow x^2 = 1$, λ ex. Dostávám body $(\pm 1, 0)$

c) $x \neq 0$, $y \neq 0$, pak z 2. a 3. rovnice:

$$1 - 2x^2 - 14y^2 = 2\lambda$$
$$7 - 7y^2 - x^2 = 8\lambda$$
$$x^2 + 4y^2 = 1$$

$$-1 - 6y^2 = 2\lambda$$
$$6 - 3y^2 = 8\lambda$$

$\Rightarrow 10 + 21y^2 = 0$

$y^2 = -\frac{10}{21}$ ↯

Kandidáti jsou: $(0,0)$, $\left(\pm\frac{1}{\sqrt{2}},0\right)$, $\left(0,\pm\frac{1}{2}\right)$, $(\pm 1,0)$

$f(0,0) = 0$ Min

$f\left(\pm\frac{1}{\sqrt{2}},0\right) = \frac{1}{2}e^{-1} = a$

$f\left(0,\pm\frac{1}{2}\right) = \frac{7}{4}e^{-\frac{1}{2}} = b$

$f(\pm 1,0) = 1e^{-2} = c$

Max.

$\frac{a}{c} = \frac{1}{2}e > 1 \Rightarrow a > c$

$\frac{b}{a} = \frac{7}{2}e^{\frac{1}{2}} > 1 \Rightarrow b > a$

$\Rightarrow b > a > c > 0$

2)

Nalezněte glob. ext. fnc M, kde

$f(x,y) = -y^2 + x^2 + \frac{4}{3}x^3$, $M = \{\underbrace{x^2+y^2 \le 4}_{g(x,y)}, \underbrace{x \le 0}_{h(x,y)}\}$

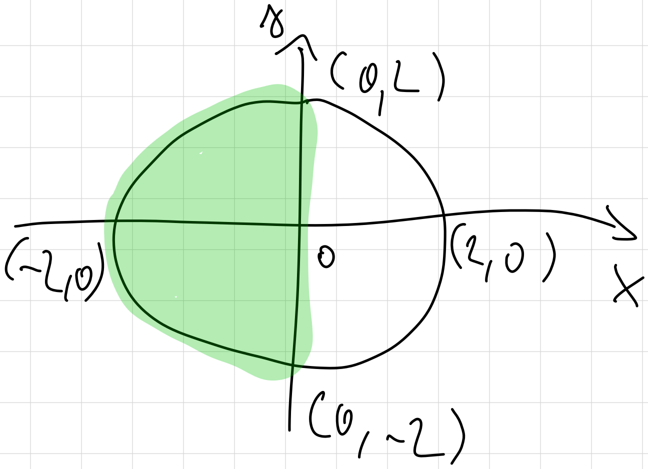
$M = M_1 \cup M_2 \cup M_3 \cup M_4$, kde

$M_1 = \{g < 4, h < 0\}$

$M_2 = \{g < 4, h = 0\}$

$M_3 = \{g = 4, h < 0\}$

$M_4 = \{g = 4, h = 0\}$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(-2,0)
0
(2,0)
x
(0,-2)