Určete extrémy f na $\mathbb{R}^2$.

$$f(x,y) = 2x^3 + 2y^3 + 3x^2y^2$$

$$\frac{\partial f}{\partial x}(x,y) = 6x^2 + 6xy^2, \quad \frac{\partial f}{\partial y}(x,y) = 6y^2 + 6yx^2$$

$$\frac{\partial^2 f}{\partial x^2}(x,y) = 12x + 6y^2, \quad \frac{\partial^2 f}{\partial y^2}(x,y) = 12y + 6x^2$$

$$\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{\partial^2 f}{\partial y \partial x}(x,y) = 12xy$$

$$\nabla f(x,y) = (0,0) \iff 6x(x+y^2) = 6y(y+x^2) = 0$$

$x = y = 0$ nebo $x(1+x^3) = 0 \iff x = -1$, $y = -x^2$ $\qquad y = -1$

$$D^{(2)} f(-1,-1) = \begin{pmatrix} -6 & 12 \\ 12 & -6 \end{pmatrix}$$ což je indefinitní

$\Rightarrow (-1,-1)$ je sedlový bod.

$D^{(2)} f(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ což je pozitivně/negativně semidefinitní, i.e. Věta 7.5 nerídá nic.

Fix $y=0$. Pak $f(x,0) = 2x^3$, i.e. f nemá v bodě $(0,0)$ extrém.