mmma 101 − 20.10.

$$\lim_{n\to+\infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}} = \lim_{n\to+\infty} \frac{3^n\left(\left(-\frac{2}{3}\right)^n + 1\right)}{3^{n+1}\left(\left(-\frac{2}{3}\right)^{n+1} + 1\right)} = \frac{1}{3}$$

$$\lim_{n\to+\infty} \sqrt[n]{n^2 + 2^n + 3^n} = \lim_{n\to+\infty} \sqrt[n]{3^n\left(\frac{n^2}{3^n} + \left(\frac{2}{3}\right)^n + 1\right)}$$

$$= 3 \lim_{n\to+\infty} \sqrt[n]{\frac{n^2}{3^n} + \left(\frac{2}{3}\right)^n + 1}$$

$$1 = \sqrt[n]{1} \leq \sqrt[n]{\frac{n^2}{3^n} + \left(\frac{2}{3}\right)^n + 1} \leq \sqrt[n]{2} \xrightarrow{n\to+\infty} 1$$

$\downarrow n\to+\infty$ $\downarrow$ z Vetjo

1   1 policajked ~~pro~~ $\exists n_0 \in \mathbb{N}, \forall n > n_0$

necomol plali!

$$\left(\lim_{n\to+\infty} \frac{n^2}{3^n} + \left(\frac{2}{3}\right)^n = 0 \Rightarrow \text{pro } \varepsilon = 1, \text{ ex. } n_0 \in \mathbb{N},\right.$$

$$\left.\text{pro } n > n_0 : \overset{-1}{0} < \frac{n^2}{3^n} + \left(\frac{2}{3}\right)^n < 1\right)$$

• $\lim\limits_{n\to+\infty} a_n = L \in \mathbb{R} \overset{df}{\Leftrightarrow}$

$\forall \varepsilon>0,\ \exists n_0\in\mathbb{N},\ \forall n\in\mathbb{N}, n>n_0 :\ a_n \in (L-\varepsilon, L+\varepsilon)$

• pro $A, B \in \mathbb{R};\ n \in \mathbb{N}$:

$$A^n - B^n = (A-B)\Big(\sum_{j=0}^{n-1} A^{n-1-j} B^j\Big)$$
$$= (A-B)\left(A^{n-1} + A^{n-2}B + \dots + AB^{n-2} + B^{n-1}\right)$$

$$\lim_{n\to+\infty} \underbrace{\sqrt[3]{n+1}}_{\substack{\parallel \\ A}} - \underbrace{\sqrt[3]{n}}_{\substack{\parallel \\ B}} = \lim_{n\to+\infty} \frac{\cancel{n}+1-\cancel{n}}{\left(\sqrt[3]{n+1}\right)^2 + \sqrt[3]{n+1}\cdot\sqrt[3]{n} + \left(\sqrt[3]{n}\right)^2} =$$

rozšíříme A^2+AB+B^2

$$\lim_{n\to+\infty} \underbrace{\frac{1}{n^{2/3}}}_{\to 0} \cdot \frac{1}{\underbrace{\underbrace{\left(\frac{n+1}{n}\right)^{2/3}}_{\to 1} + \underbrace{\left(\frac{n+1}{n}\cdot\frac{n}{n}\right)^{1/3}}_{\to 1} + 1}_{\to 3}} \overset{AL}{=} 0$$

Fakt: Pokud $\{a_n\}$ posl. $\lim\limits_{n\to+\infty} a_n \geq 0$ a $\forall n\in\mathbb{N};\ a_n \geq 0$

$\alpha \in \mathbb{R}$. Pak $\lim\limits_{n\to+\infty} (a_n)^\alpha = \left(\lim\limits_{n\to+\infty} a_n\right)^\alpha$

$$\lim_{n \to +\infty} (-1)^n \sqrt{n}\left(\sqrt{n+1} - \sqrt{n}\right) =$$

rozšiř $\sqrt{n+1} + \sqrt{n}$

$$\lim_{n \to +\infty} (-1)^n \sqrt{n}\, \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \to +\infty} (-1)^n \frac{\sqrt{n}}{\sqrt{n}\left(\sqrt{\frac{n+1}{n}} + 1\right)}$$

vytknu největší člen

nelze použít AL

ale: $\lim_{n \to +\infty} \underbrace{\sqrt{n}\left(\sqrt{n+1} - \sqrt{n}\right)}_{a_n} = \frac{1}{2}$

$\Rightarrow \forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N},\ \forall n > n_0 : a_n \in \left(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right)$

Vol $\varepsilon = \frac{1}{4}$: $\exists n_0 \in \mathbb{N},\ \forall n \geq n_0 : a_n \in \left(\frac{1}{4}, \frac{3}{4}\right)$

$\Rightarrow \lim_{n \to +\infty} a_n (-1)^n$ neexistuje

(Kdyby ex. $a = L \in \mathbb{R}$, pak volím $\varepsilon > 0$ tak, aby $(L - \varepsilon, L + \varepsilon) \cap \left(\frac{1}{4}, \frac{3}{4}\right) = \emptyset$ nebo $(L - \varepsilon, L + \varepsilon) \cap \left(-\frac{3}{4}, -\frac{1}{4}\right) = \emptyset$

Protože $(-1)^n a_n \in \left(\frac{1}{4}, \frac{3}{4}\right)$ pro n sudé, $n > n_0$

$(-1)^n a_n \in \left(-\frac{3}{4}, -\frac{1}{4}\right)$ n liché, $n > n_0$,

nelze splnit ~~definici limity pro $a_n (-1)^n$~~.

$\exists n_1 \in \mathbb{N},\ \forall n > n_1 : a_n \in (L - \varepsilon, L + \varepsilon)$.

$$\lim_{n\to+\infty} \underset{A}{\sqrt[3]{n^3+1}} - \underset{B}{\sqrt[2]{n^2+1}} =$$

násobíme $A^5 + A^4B + A^3B^2 + A^2B^3 + AB^4 + B^5$

$$\lim_{n\to+\infty} \frac{(n^3+1)^2 - (n^2+1)^3}{\left(\sqrt[3]{n^3+1}\right)^5 + \dots + \left(\sqrt[2]{n^2+1}\right)^5} = \lim_{n\to+\infty} \frac{\cancel{n^6} + 2n^3 + 1 - (\cancel{n^6} + 3n^4 + 3n^2 + 1)}{n^5\underbrace{(\dots\dots\dots)}_{\longrightarrow 6}}$$

$$= \lim_{n\to+\infty} \frac{n^4\overbrace{\left(\frac{2}{n} + \frac{1}{n^4} - \frac{3}{\cancel{n}} - \frac{3}{n^2} - \frac{1}{n^4}\right)}^{\longrightarrow -3}}{n^5\underbrace{(\qquad\qquad)}_{\longrightarrow 6}} = 0$$

např.: $\dfrac{A^4B^1}{n^5} = \dfrac{\left(\sqrt[3]{n^3+1}\right)^4}{n^4} \cdot \dfrac{\sqrt[2]{n^2+1}}{n} =$

$$= \left(\sqrt[3]{1+\frac{1}{n^3}}\right)^4 \cdot \sqrt{1+\frac{1}{n^2}} \xrightarrow{n\to+\infty} 1$$