CLENSHAW'S ALGORITMUS

[B: Numerical Recipes (2007)] p.176

- Uvažujme polynomy $F_i(x)$ splňující 3-člennou rekurenci
$$F_{k+1}(x) = \alpha(x) F_k(x) - \beta F_{k-1}(x), \qquad k = 1, \dots, n-1$$
$F_0(x)$, $F_1(x)$ dáno
$\alpha(x)$ dáno, β dáno.

- Chceme vyhodnotit sumu
$$f(x) = \sum_{k=0}^{n} c_k F_k(x),$$
c_k jsou dané koeficienty.

- Definujme posloupnost
$$y_{n+2} = y_{n+1} = 0$$
$$y_j = \alpha(x) y_{j+1} - \beta y_{j+2} + c_j, \qquad j = n, n-1, \dots, 1.$$

- Uvědomme si, že platí
$$F_{k+1}(x) - \alpha(x) F_k(x) + \beta F_{k-1}(x) = 0, \qquad k = 1, \dots, n-1,$$
$$c_j = y_j - \alpha(x) y_{j+1} + \beta y_{j+2}, \qquad j = n, \dots, 1.$$

- Platí
$$f(x) = \sum_{k=0}^{n} c_k F_k(x) = \sum_{j=1}^{n} [y_j - \alpha(x) y_{j+1} + \beta y_{j+2}] F_j + c_0 F_0$$
$$= [y_n - \cancel{\alpha(x) y_{n+1}} + \cancel{\beta y_{n+2}}] F_n +$$
$$[y_{n-1} - \alpha(x) y_n + \cancel{\beta y_{n+1}}] F_{n-1} +$$
$$[y_{n-2} - \alpha(x) y_{n-1} + \beta y_n] F_n +$$
$$\vdots$$
$$[y_2 - \alpha(x) y_3 + \beta y_4] F_2 +$$
$$[y_1 - \alpha(x) y_2 + \beta y_3] F_1 +$$
$$[c_0 - \beta y_2 + \beta y_2] F_0$$

vytkneme y_j, $j = n, \dots, 2$

$$= \sum_{j=2}^{n} y_j \overbrace{[F_j(x) - \alpha(x) F_{j-1}(x) + \beta F_{j-2}(x)]}^{0}$$
$$+ c_0 F_0(x) - \beta y_2 F_0(x) + y_1 F_1(x).$$

- Speciálně, pro Čebyš. pol. je $\alpha(x) = 2x$, $\beta = 1$ a
$$\sum_{k=0}^{n} c_k T_k(x)$$
lze spočíst pomocí
$$y_{n+2} = y_{n+1} = 0, \quad y_j = 2x y_{j+1} - y_{j+2} + c_j, \quad j = n, \dots, 1$$
$$f(x) = c_0 - y_2 + y_1 x.$$
$$f(x) = \tfrac{1}{2}[c_0 + y_0 - y_2].$$

Dodefinujeme-li $y_0 = 2x y_1 - y_2 + c_0$, je $x y_1 = \frac{1}{2}[y_0 + y_2 - c_0]$ a

- Pro $F_k = x^k$, tj $F_k = x F_{k-1}$, $\alpha(x) = x$, $\beta = 0$
  dostaneme klasické Hornerovo schéma
  $y_{n+2} = y_{n+1} = 0$, $y_j := x y_{j+1} + c_j$, $j = n, \dots, 1$
  $$f(x) = x y_1 + c_0 .$$