1) Vyšetřit lokální extrémy f v $\mathbb{R}^2$

kde $f(x,y) = x^2 + 4x + 5 - 3y^2 - 3y$

$\frac{\partial f}{\partial x}(x,y) = 2x + 4$

$\frac{\partial f}{\partial y}(x,y) = -6y - 3$

$\nabla f(x,y) = 0$

$\Updownarrow$

$x = -2$

$y = -\frac{1}{2}$

$\frac{\partial^2 f}{\partial x^2}(x,y) = 2, \frac{\partial^2 f}{\partial y^2}(x,y) = -6$

$$\frac{\partial^2 f}{\partial x \partial y}(x,y) = 0 \qquad \text{Tedy } \nabla^2 f(x,y) = \begin{pmatrix} 2 & 0 \\ 0 & -6 \end{pmatrix}$$

A je indef. $\Leftrightarrow$ $a_{11} a_{22} < a_{12}^2$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}$$

$-12 < 0$

Matice $\nabla^2 f(-2, -\frac{1}{2})$ je indefinitní, bod je sedlovým bodem f.

(2) $f(x,y) = (x^2+5y^2)\, e^{-(3x^2+y^2)}$, на $\mathbb{R}^2$.

$$\frac{\partial f}{\partial x}(x,y) = 2x\, e^{-(3x^2+y^2)} + (x^2+5y^2)\, e^{-(3x^2+y^2)}(-6x)$$

$$\frac{\partial f}{\partial y}(x,y) = 10y\, e^{-(3x^2+y^2)} + (x^2+5y^2)\, e^{-(3x^2+y^2)}(-2y)$$

$$\frac{\partial^2 f}{\partial x^2}(x,y) = 2e^{-(3x^2+y^2)} + 2x\,e^{-(3x^2+y^2)}(-6x)$$
$$+ (-18x^2 - 30y^2)\,e^{-(3x^2+y^2)} +$$
$$+ (-6x^3 - 30y^2x)\,e^{-(3x^2+y^2)}(-6x)$$

$$\frac{\partial^2 f}{\partial y^2}(x,y) = 10e^{-(3x^2+y^2)} + 10y\,e^{-(3y^2+y^2)}(-2y)$$
$$+ (-2x^2 - 30y^2)\,e^{-(3x^2+y^2)}$$
$$+ (-2x^2y - 10y^3)\,e^{-(3x^2+y^2)}(-2y)$$

$$\frac{\partial^2 f}{\partial x \partial y}(x,y) = 10y\, e^{-(3x^2+y^2)}(-6x) +$$

$$+(-2y)(2x)\, e^{-(3x^2+y^2)} +$$

$$+(-2y)(x^2+5y^2)\, e^{-(3x^2+y^2)}(-6x)$$

$$\frac{\partial f}{\partial x}(x,y) = e^{-(3x^2+y^2)}\, x \, \Big(\underbrace{-6x^2 - 3y^2 + 2}_{(*)}\Big)$$

$$\frac{\partial f}{\partial y}(x,y) = \underset{\neq 0}{e^{-(3x^2+y^2)}}\, y \, \Big(\underbrace{-2x^2 - 10y^2 + 10}_{(**)}\Big)$$

Pokud je $(*) = 0$ pak $(**) \neq 0$

a naopak.

Kritické body jsou:

1) $x = y = 0$

2) $x = 0$, $y^2 = 1$, i.e. body $(0,1)$, $(0,-1)$

3) $y = 0$, $x^2 = \frac{1}{3}$, i.e. body $\left(\frac{1}{\sqrt{3}}, 0\right)$, $\left(-\frac{1}{\sqrt{3}}, 0\right)$

1) $x = y = 0$: $D^2 f(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & 10 \end{pmatrix}$

což je PD $\Rightarrow$ $(0,0)$ je ostré lok min.

2) $x = 0,\ y^2 = 1$: $D^2 f(x,y) = \begin{pmatrix} -28e^{-1} & 0 \\ 0 & -20e^{-1} \end{pmatrix}$

což je ND $\Rightarrow$ $(0,1)$, $(0,-1)$ jsou ostré lok. max.

3) $y = 0$, $x^2 = \frac{1}{3}$, $D^2 f(x,y) = \begin{pmatrix} -4e^{-1} & 0 \\ 0 & \left(10 - \frac{2}{3}\right)e^{-1} \end{pmatrix}$

CoE je indefinitní, body $\left(\frac{1}{\sqrt{3}}, 0\right)$, $\left(-\frac{1}{\sqrt{3}}, 0\right)$ jsou sedlové.

Na $\mathbb{R}^2$: 1) $f(x,y) = 3x + 5 - 2x^2 - y^2$

2) $f(x,y) = (x^2+y^2)\, e^{-(x^2+y^2)}$

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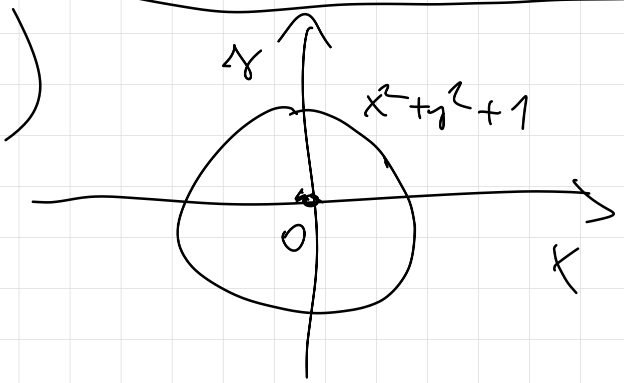
2)



v bodech $\{x^2+y^2=1\}$

je $\nabla^2 f(x,y)$ NSD

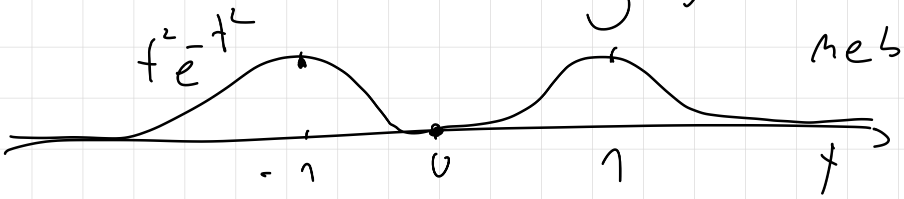
$$\begin{pmatrix} -4e^{-1}x^2 & -4e^{-1}xy \\ -4e^{-1}xy & -4e^{-1}y^2 \end{pmatrix}$$

V tomto případě je v bodec

$\{x^2+y^2=1\}$ lok max (ne ostré)

$(x^2+y^2)\,e^{-(x^2+y^2)}$ závisí jen na x^2+y^2,

i.e. stačí vyšetřit $t^2 e^{-t^2}$ na $(-\infty,\infty)$



nebo te^{-t} na $[0,\infty)$