$(*)$ $f(x,y) = 2x^4y + x^3 + y^3 + xy - 1$, $\Pi = (1,0)$

$f \in C^\infty(\mathbb{R}^2)$

$f(1,0) = 0 + 1 + 0 + 0 - 1$

$\frac{\partial f}{\partial y}(x,y) = 2x^4 + 3y^2 + x \Rightarrow \frac{\partial f}{\partial y}(1,0) = 3 \neq 0$

Tedy podle VoIF rovnice (*) definuje na okolí bodu 1 funkci φ třídy $\mathcal{C}^\infty$.

$$2x^4 \varphi(x)' + 8x^3 \varphi(x) + 3x^2 + 3\varphi(x)^2 \varphi'(x) + \varphi(x) + x\varphi'(x) = 0$$

i.e.

$$2\varphi'(1) + 0 + 3 + 0 + 0 + \varphi'(1) = 0$$

$$\varphi'(1) = -1$$

$$2x^4 \varphi''(x) + 8x^3 \varphi'(x) + 8x^3 \varphi'(x) + 24x^2 \varphi(x)$$

$$+ 6x + 3\varphi^2(x)\, \varphi''(x) + 6\varphi(x)\, \varphi'(x)^2 + \varphi'(x)$$

$$+ \varphi'(x) + x\varphi''(x) = 0 \quad \text{i.e.}$$

$$2\varphi''(1) - 8 - 8 + 0 + 6 + 0 + 0 - 1 - 1 + \varphi''(1) = 0$$

$$\varphi''(1) = 4$$