$$\begin{matrix} 1 + 2 + \dots + n \\ n + (n-1) + \dots + 1 \end{matrix} + \quad = (n+1)n$$

$$1 + 2 + \dots + n = \frac{(n+1)n}{2}$$

$$\lim_{n\to+\infty}\frac{n^2+n}{2n^2-7}=\lim_{n\to+\infty}\frac{\cancel{n^2}\left(1+\frac{1}{n}\right)}{\cancel{n^2}\left(2-\frac{7}{n^2}\right)}$$

$$=\lim_{n\to+\infty}\frac{1+\underbrace{\tfrac{1}{n}}_{\xrightarrow{n\to+\infty}0}}{2-\underbrace{\tfrac{7}{n^2}}_{\to 0}}$$

$1+\frac{1}{n} \xrightarrow{n\to+\infty} 1$ (limita součtu)

$2-\frac{7}{n^2} \longrightarrow 2$

lim. podílu

$$\overset{\downarrow}{=}\ \frac{1}{2}$$

Př. Součet $1+a+\dots+a^n$, $n\in\mathbb{N}$

1) Ukážeme $A^{n+1}-B^{n+1}=(A-B)\underbrace{(A^n+A^{n-1}B+\dots+AB^{n-1}+B^n)}_{\sum\limits_{k=0}^{n}A^{n-k}B^k}$

$$(A-B)\left(\sum_{k=0}^{n}A^{n-k}B^k\right)=\sum_{k=0}^{n}A^{n-k+1}B^k-\sum_{k=0}^{n}A^{n-k-1+1}B^{k+1}$$

$k+1=l$

$$=\sum_{k=0}^{n}A^{n-k+1}B^k-\sum_{l=1}^{n+1}A^{n-l+1}B^l=A^{n+1}-B^{n+1}\ \checkmark$$

2) Volme $A=a$; $B=1$:

$$a^{n+1}-1=(a-1)(a^n+a^{n-1}+\dots+a+1)$$

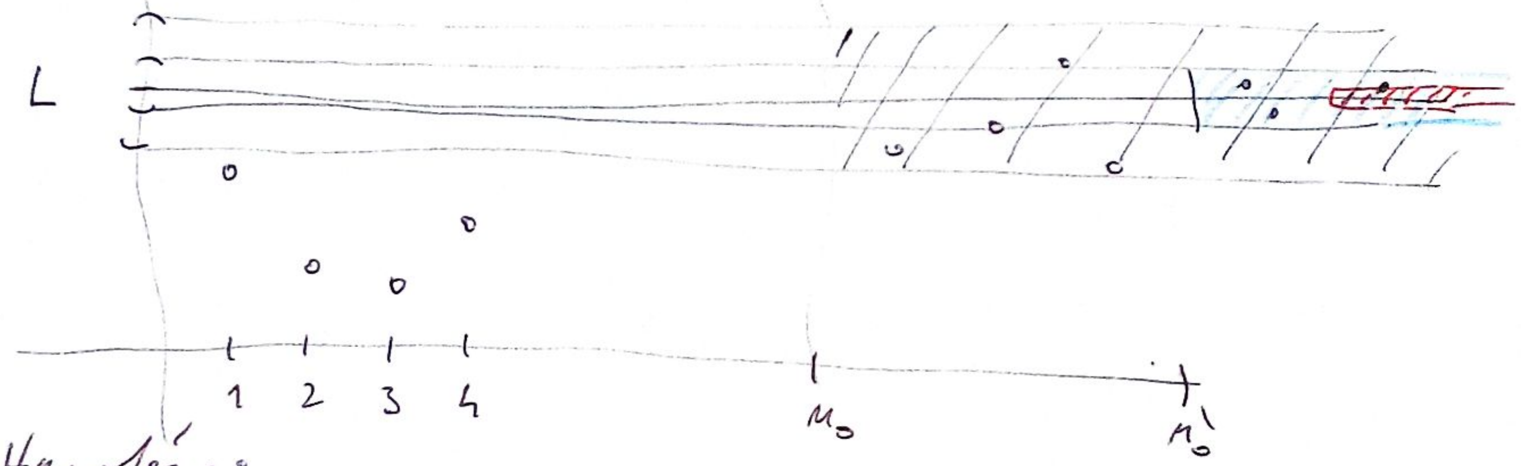
$$\frac{a^{n+1}-1}{a-1}=a^n+a^{n-1}+\dots+a+1 \text{ pro } a\in\mathbb{R}\setminus\{1\}$$

$\mathbb{C}\setminus\{1\}$

Limity posloupností

$\{a_n\}_{n=1}^{+\infty}$, $L \in \mathbb{R}$: $\lim\limits_{n\to+\infty} a_n = L \overset{df}{\iff}$

$$\forall \varepsilon > 0, \exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n > n_0 : a_n \in (L-\varepsilon, L+\varepsilon)$$



Hypotéza:

$$\lim_{n\to+\infty} \frac{1}{\sqrt{n}} = 0,$$

Dk: Fix $\varepsilon > 0$ lib. Hledám $n_0 \in \mathbb{N}$. $\forall n > n_0, n \in \mathbb{N}$

$$\underset{\checkmark}{-\varepsilon <} \frac{1}{\sqrt{n}} < \varepsilon, \quad \frac{1}{\varepsilon} < \sqrt{n}, \quad \frac{1}{\varepsilon^2} < n$$

Z Archimedovy vl. $\exists\, n_0 \in \mathbb{N}$, $n_0 > \frac{1}{\varepsilon^2}$

$$\Rightarrow \forall n > n_0 : \frac{1}{\sqrt{n}} \in (-\varepsilon, \varepsilon) \quad \checkmark \quad \perp$$

Z věty o aritmetice limit: $\forall k \in \mathbb{N}$:

$$\lim_{n\to+\infty} \frac{1}{n^k} = 0 \quad (\text{Indukce dle } k \in \mathbb{N}.)$$

Cvičení mma 101 – 12. 10.

Opak. sup a inf:

$$M_5 = \left\{ \frac{p}{p+q} ; p, q \in \mathbb{N} \right\}$$

$\boxed{\sup M_5 = 1}$, $\inf M_5 = 0$

Dk: 1) Chci $\forall x \in M_5 : x \leq 1$.

tj. $\frac{p}{p+q} \leq 1$, $\cancel{p} \leq p + q$; $q \geq 0$ ✓

2) $\forall s' < 1, \exists x \in M_5 ; x > s'$

$$\left(\begin{array}{l} \text{Vol } q = 1, \ \frac{p}{p+1} = s' ; \ p = ps' + s' \\ p = \frac{s'}{1-s'} + 1 \notin \mathbb{N} \text{ tj. } \frac{p}{p+1} \notin M_5 \end{array} \right)$$

Chci? $p > \frac{s'}{1-s'}$ a $p \in \mathbb{N}$. Ex. $p \in \mathbb{N}$ plyne z Archimedovy vl.