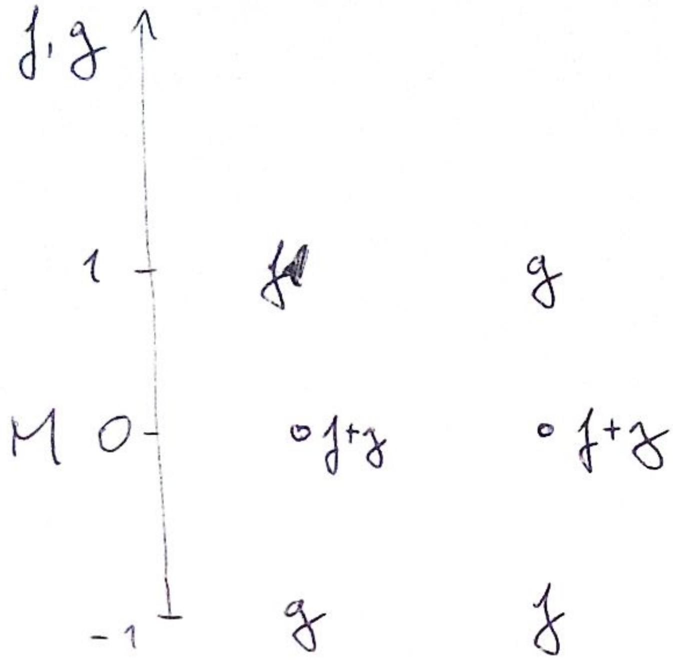
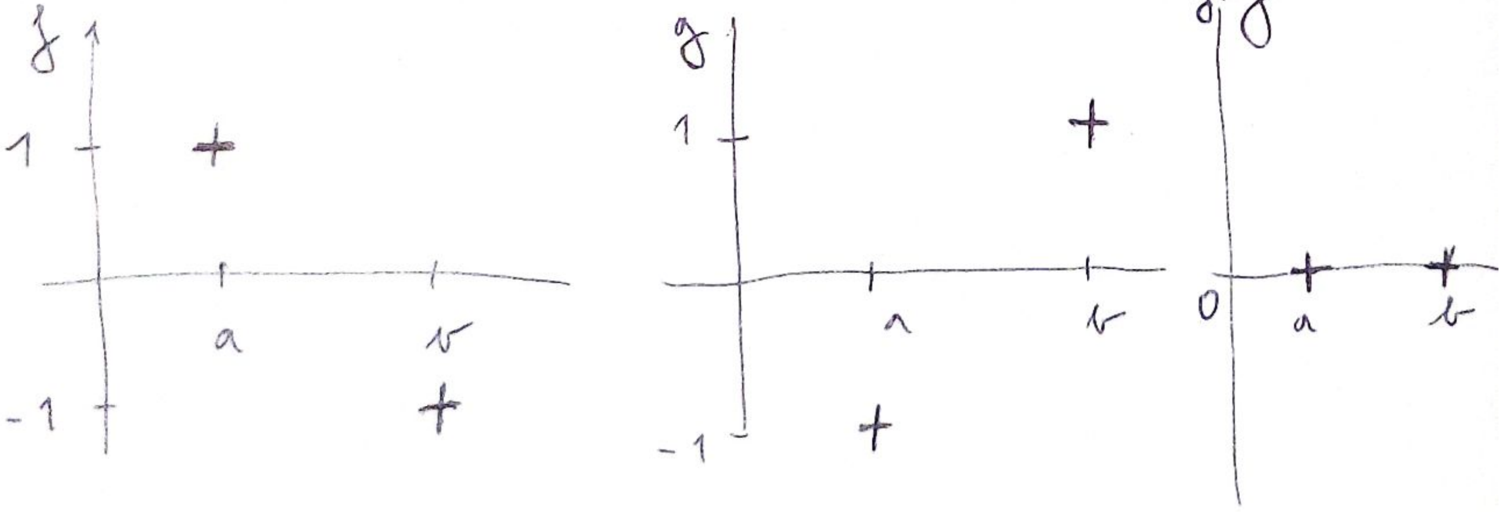


$$M = \{a, b\} \; ; \quad f(a) = 1 = g(b)$$
$$f(b) = -1 = g(a)$$



PaE $\sup (f+g)(M) = 0 < \sup f(M) + \sup g(M)$
$= 1 + 1 = 2$

Hypotéza: $\inf M_1 = 0$ Dk:

1) $\forall n \in \mathbb{N} : \frac{1}{n} \geq 0$ ✓

2) Fix $\varepsilon > 0$, hledám $x \in M : x < \varepsilon$.

$\frac{1}{n} < \varepsilon$; $n > \frac{1}{\varepsilon}$ a takové n ex. z Archimédovy vlastnosti ✓.

Př: $A, B \subset \mathbb{R}$, $s_A = \sup A$; $s_B = \sup B$

Lze něco říci o $\sup(A \cup B)$?

Hypotéza: $\sup(A \cup B) = \max(s_A, s_B) =: s$

Dk: Pp. $s = s_A$.

1) $\forall a \in A : a \leq s_A$, protože s_A je $\sup A$

$\forall b \in B : b \leq s_B \leq s_A$

$s_B = \sup B$

$\Rightarrow \forall x \in A \cup B : x \leq s$

2) $\forall s' < s, \exists x \in A \cup B : s' < x$?

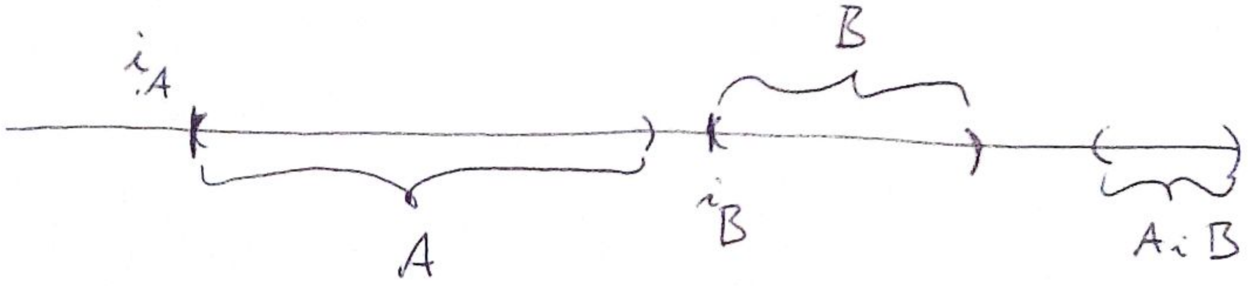
Fix $s' < s$, hledám $A \cup B \ni x$.

$s = \sup A$, z 2. vl. $\sup A$ $\exists a \in A : s' < a$

Stačí volit $x = a \in A \cup B$ ✓

Př: $A, B \subset \mathbb{R}$; $i_A = \inf A$, $i_B = \inf B$

Lze něco říci o $\inf(A \cap B)$?



$A := \{0, 2\}$ $\quad i_A = 0$

$B := \{1, 2\}$ $\quad i_B = 1$

$A \cap B = \{2\}$ $\quad \inf(A \cap B) = 2$

Obecně ne!

Př: M neprázdná, $f, g : M \to \mathbb{R}$ shora omezené

$$\sup (f+g)(M) \leq \sup f(M) + \sup g(M)$$

Dk: Stačí ukázat: $\forall y \in (f+g)(M)$:

$$y \leq \sup f(M) + \sup g(M).$$

Fix $y \in (f+g)(M)$, $\exists x \in M : (f+g)(x) = y$.

$$y = (f+g)(x) = \underbrace{f(x)}_{\in f(M)} + \underbrace{g(x)}_{\in g(M)} \leq \sup f(M) + \sup g(M)$$

✓

Cvičení 5 - sup, inf

Opakování: $M \subset \mathbb{R}$, omezená shora a zdola

definujeme sup M následovně:

$S \in \mathbb{R}$ je sup M, pokud

1) $\forall x \in M: x \leq S$

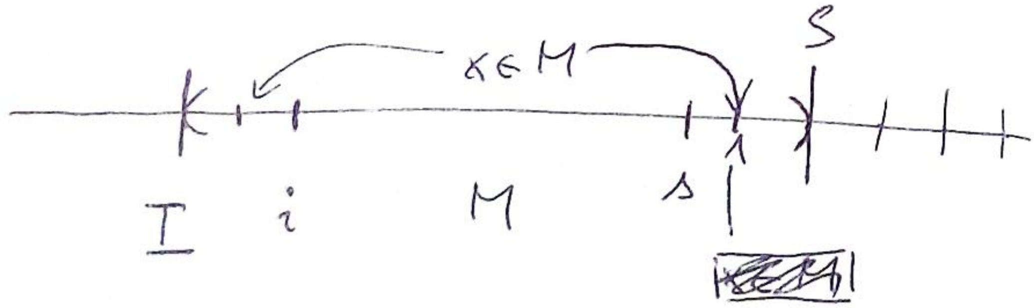
2) $\forall s < S, \exists x \in M: s < x$

inf M definujeme:

$I \in \mathbb{R}$ je inf M, pokud

1) $\forall x \in M: I \leq x$

2) $\forall i > I, \exists x \in M: x < i$



1) $M_1 = \left\{ \frac{1}{n}; n \in \mathbb{N} \right\}$

Hypotéza: $\sup M_1 = 1$ Dk: 1) $\forall n \in \mathbb{N}: \frac{1}{n} \leq 1$ ✓

2) Fix $s < 1$, hledám $x \in M: s < x$. Stačí $x = 1$ ✓