Otevřenost / uzavřenost množin

Kritérium:

f je spojitá $\mathbb{R}^n \to \mathbb{R}$, pak

$\{f > c\}$, $\{f < c\}$ je ot.

$\{f \geq c\}$, $\{f \leq c\}$, $\{f = c\}$ je uz.

$A = \{(x,y) \in \mathbb{R}^2 \mid x^2 + e^y > 17\}$

- Projekce jsou spojité
- Složení spoj. je spoj.
- Součet spoj. je spoj.

$\Rightarrow$ $x^2 + e^y$ je spojitá $\Rightarrow$ A je ot.

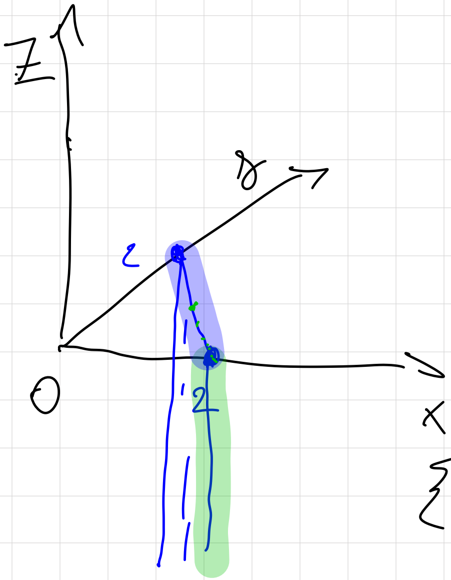
$$B = \{(x,y) \in \mathbb{R}^2 \mid |x+y| > x+y\}$$

$$|x+y| > x+y \iff |x+y| - x - y > 0$$

- Projekce jsou spojité
- Součet/rozdíl spoj. je spoj.
- Složení spoj. je spoj.

$\Rightarrow$ B je ot.

$$C = \{(x,y,z) \in \mathbb{R}^3 \mid x \geq 0,\ y > 0,\ x+y = 2,\ z \leq 0\}$$



C není ot., neboť je pod. roviny a rovina v $\mathbb{R}^3$ má prázdný vnitřek

C není uz., neboť

$$\underbrace{\left\{\left(2-\frac{1}{n}, \frac{1}{n}, 0\right)\right\}_{n \in \mathbb{N},\ n \neq 0}}_{\in C} \xrightarrow{n \to \infty} (2,0,0) \notin C$$

Rovina má prázdný vnitřek:

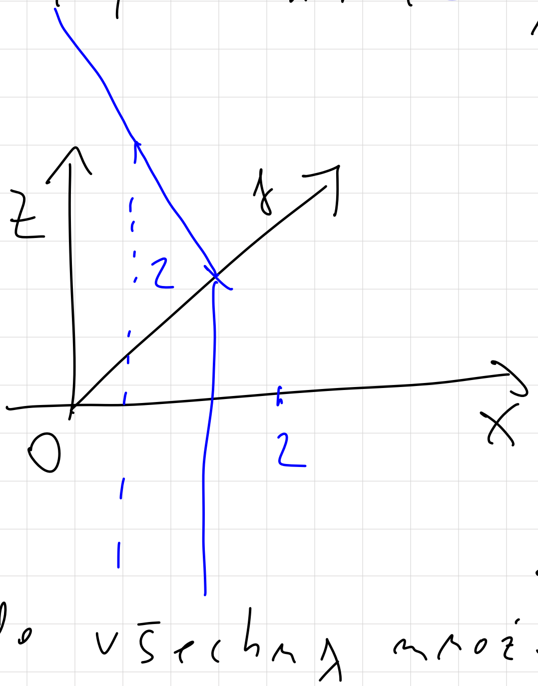
Je-li $\rho = \{(x,y,z) \in \mathbb{R}^3 \mid ax + by + cz + d = 0\}$

pak $n = (a, b, c)$ je kolmý na ρ.

Nechť $\vec{x}$, $\varepsilon > 0$, pak $x + \frac{\varepsilon}{2} \cdot \frac{n}{\|n\|} \in B(x, \varepsilon)$

ale $x + \frac{\varepsilon}{2} \frac{n}{\|n\|} \notin \rho$. $\square$

$D=\{(x,y,z)\in\mathbb{R}^3 \mid x\le 0,\ y>0,\ x+y=2,\ z\le 0\}$



$y = 2-x \ge 2 > 0$

● je navíc.

Tedy $D = \{x\le 0,\ z\le 0,\ x+y=2\} =$

$= \{x\le 0\}\cap\{z\le 0\}\cap\{x+y=2\}$

kde všechny množiny na RHS jsou uz.

$\Rightarrow$ D je uz., vnitřek je prázdný, vizte výše.

Parciální derivace:

$f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x,y,z) = xy + yz + zx$

$$\frac{\partial f}{\partial x}(x,y,z) = y + 0 + z$$

$$\frac{\partial f}{\partial y}(x,y,z) = x + z + 0$$

$$\frac{\partial f}{\partial z}(x,y,z) = 0 + y + x$$

na $\mathbb{R}^3$

$$f : \mathbb{R}^2 \to \mathbb{R} \quad , \quad f(x, y) = e^{xy}$$

$$\frac{\partial f}{\partial x}(x, y) = e^{xy} \cdot y$$

$$\frac{\partial f}{\partial y}(x, y) = e^{xy} \cdot x$$

$f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = |x| \cdot |y|$

1) $x \neq 0$, $y \neq 0$

$$\frac{\partial f}{\partial x}(x,y) = |y| \operatorname{sgn}(x)$$

$$\frac{\partial f}{\partial y}(x,y) = |x| \operatorname{sgn}(y)$$

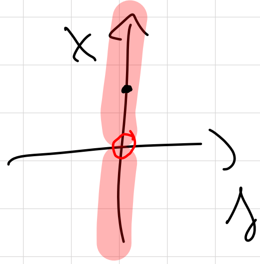
2) $x \neq 0$, $y = 0$

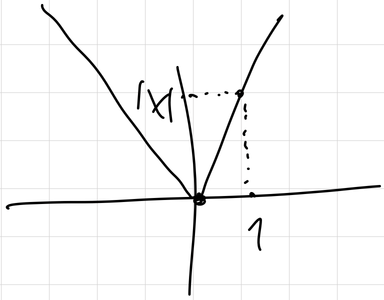
pak $f(x,0) = |x| \cdot 0$, tedy

$$\frac{\partial f}{\partial x}(x,0) = 0$$

ale

$$\frac{\partial f}{\partial y}(x,0) \text{ neex.}$$





3) $x = 0, y \neq 0$ pak

$$\frac{\partial f}{\partial y}(0,y) = 0$$

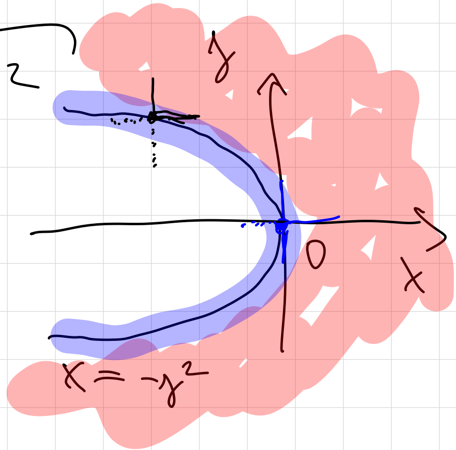
$$\frac{\partial f}{\partial x}(0,y) \text{ neex.}$$, jako výše.

4) $x = y = 0$ tada

$f(x, 0) = 0 = f(y, 0)$, tedy

$$\frac{\partial f}{\partial x}(0,0) = 0 = \frac{\partial f}{\partial y}(0,0) = 0$$

$f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = \sqrt{x+y^2}$



$D_f = \{(x,y) \in \mathbb{R}^2 \mid x+y^2 \geq 0\}$

1) $x+y^2 > 0$, pak

$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{2}\frac{1}{\sqrt{x+y^2}} \cdot 1$$

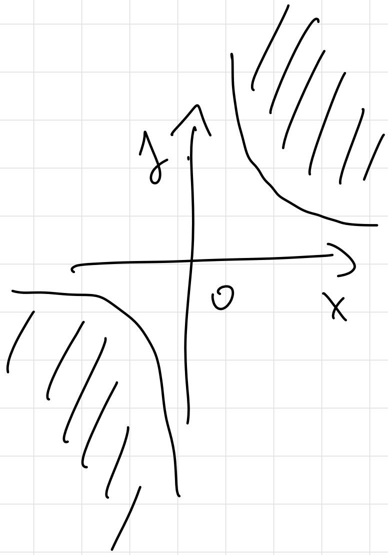
$$\frac{\partial f}{\partial y}(x,y) = \frac{1}{2}\frac{1}{\sqrt{x+y^2}} \cdot 2y$$

2) $x = y = 0$

$f(0,y) = \sqrt{y^2} = |y|$ a tedy

$\frac{\partial f}{\partial y}(0,0)$ neex.

$f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = \sqrt{e^{xy} - e}$



$D_f = \{ e^{xy} - e \geq 0 \} = \{ xy \geq 1 \}$

exp. je rostoucí

$xy > 1$, grad

$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{2} \frac{1}{\sqrt{e^{xy} - e}} \cdot e^{xy} y$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{1}{2} \frac{1}{\sqrt{e^{xy} - e}} \cdot e^{xy} x$$