1a) Cvičení MA1, 29. září

$$\sqrt{x^2+2x-3} \geq \sqrt{x^2+3x-4}$$

$x^2+2x-3 \geq 0$; $\Rightarrow x \in (-\infty, -3] \cup [1, +\infty)$

$x^2+3x-4 \geq 0$ $\Rightarrow x \in (-\infty, -4] \cup [1, +\infty)$

$\cancel{x^2}+2x-3 \geq \cancel{x^2}+3x-4$

$1 \geq x$ $\Rightarrow x \in (-\infty, 1]$

$\Rightarrow x \in (-\infty, -4] \cup \{1\}$

Výroky: $\exists, \forall$ $\neg\exists \to \forall\neg$

$\neg\forall \to \exists\neg$

$V: \forall x, y \in \mathbb{R}: x^2+y^2 > 0$

$\neg V: \exists x \in \mathbb{R}, \exists y \in \mathbb{R}: x^2+y^2 \leq 0$; $x=y=0$

$V_1: \forall x \in \mathbb{R}, \exists y \in \mathbb{N}: (y \leq x) \wedge (y+1 > x)$

$\neg V_1: \exists x \in \mathbb{R}, \forall y \in \mathbb{N}: (y > x) \vee (y+1 \leq x)$

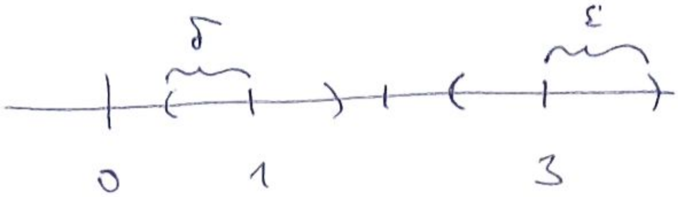
pro $x=-1$ neex. $y \in \mathbb{N}$ s $y \leq -1$

$V_2: \forall \varepsilon > 0, \exists \delta > 0, \forall x \in \mathbb{R}: 0 < |x-1| < \delta \Rightarrow |x-3| < \varepsilon$

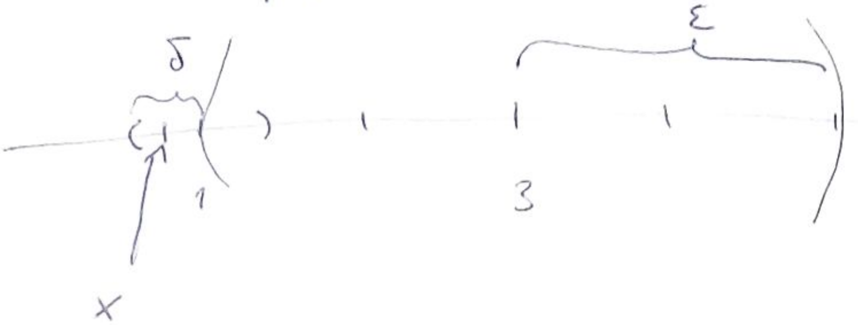
$\neg V_2: \exists \varepsilon > 0, \forall \delta > 0, \exists x \in \mathbb{R}: \neg[0 < |x-1| < \delta \Rightarrow |x-3| < \varepsilon]$

$\neg(A \Rightarrow B) \Leftrightarrow (A \wedge \neg B)$

$\exists \varepsilon > 0, \forall \delta > 0, \exists x \in \mathbb{R}: 0 < |x-1| < \delta \wedge |x-3| \geq \varepsilon$



platí $\neg V_2$: $\varepsilon = 2$, pro $\delta > 0$ vol $x = 1 - \frac{\delta}{2}$



1. $\sum_{i=1}^{n} (2i+1) = n^2$ pro $n \in \mathbb{N}$

Dk: M1: 1. krok $n = 1$: $\sum_{i=1}^{n} 2i+1 = 2+1 = 3 \neq 1$

Výrok neplatí $\perp$ pro $n = 1$

$$\sum_{i=0}^{n-1} (2i+1) = n^2$$

Dk: M1 1. krok $n = 1$: $\sum_{i=0}^{n-1} 2i+1 = 1 = n^2 = 1$

2. krok: pp. že výrok platí pro $n \in \mathbb{N}$ a ukážeme, že platí i pro $n+1$

$$\sum_{i=0}^{(n+1)-1} (2i+1) \stackrel{?}{=} (n+1)^2$$

$$\parallel$$

indukční krok

$$\sum_{i=0}^{n-1} (2i+1) + 2n+1 = n^2 + 2n + 1 = (n+1)^2$$

$\perp$

• Inverse k $f(x) = \dfrac{2\sqrt{x}}{4-\sqrt{x}}$

– $D(f)$: $x \geq 0$, $4 \neq \sqrt{x}$, $x \neq 16$

$D(f) = [0, 16) \cup (16, +\infty)$

– $f: D(f) \to H(f)$

$f_{-1}: H(f) \to D(f)$

Hledání f_{-1}, $H(f)$: řešíme $y = \dfrac{2\sqrt{x}}{4-\sqrt{x}}$ pro $y \in \mathbb{R}$

~~Pokud~~ Pokud ex. řešení $x \in D(f)$, pak $y \in H(f)$. ~~Pokud x jediné~~

$(4-\sqrt{x})y = 2\sqrt{x}$, $4y = \sqrt{x}(2+y)$,

pokud $y \neq -2$: $\sqrt{x} = \dfrac{4y}{2+y}$, $x = \left(\dfrac{4y}{2+y}\right)^2$

pokud $\dfrac{4y}{2+y} \geq 0$

$4y \geq 0 \wedge 2+y > 0 \quad \vee \quad (4y \leq 0) \wedge (2+y < 0)$

$y \geq 0 \wedge y > -2 \qquad y \leq 0 \wedge y < -2$

$y \in [0, +\infty) \qquad y \in (-\infty, -2)$

$\Rightarrow H(f) = (-\infty, -2) \cup [0, +\infty)$, $f_{-1}(y) = \left(\dfrac{4y}{2+y}\right)^2$

pro $y \in H(f)$.