Úkol 1

1) $F(z) = \lg|z| + i \operatorname{arctg}\left(\frac{\operatorname{Im} z}{\operatorname{Re} z}\right)$; $F = F_1 + iF_2$; $z = x+iy$

$F_i = F_i(x,y)$

$$\Rightarrow \quad F_1(x,y) = \lg\sqrt{x^2+y^2} = \frac{1}{2}\lg(x^2+y^2)$$

$$F_2(x,y) = \operatorname{arctg}\left(\frac{y}{x}\right)$$

$$\left.\begin{aligned}
\partial_x F_1(x,y) &= \frac{1}{2}\,\frac{1}{x^2+y^2}\cdot 2x = \frac{x}{x^2+y^2} \\
&\overset{!}{=} \\
\partial_y F_2(x,y) &= \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x} = \frac{1}{\frac{x^2+y^2}{x^2}}\cdot\frac{1}{x} = \frac{x}{x^2+y^2} \\
\partial_y F_1(x,y) &= \frac{y}{x^2+y^2} \\
&\overset{!}{=} \\
-\partial_x F_2(x,y) &= \frac{-1}{1+\frac{y^2}{x^2}}\cdot\left(-\frac{y}{x^2}\right) = \frac{y}{x^2+y^2}
\end{aligned}\right\} \text{pro } x>0$$

$\Rightarrow$ F splňuje Cauchy-Riemannovy podmínky $\Rightarrow$ má derivaci na $\{\operatorname{Re} z > 0\}$.

2) $F(z) = z^k$, $\gamma(t) = e^{it}$; $t \in [0, 2\pi]$; $\gamma'(t) = ie^{it}$

$$\int_\gamma F = \int_0^{2\pi} (e^{it})^k \cdot i e^{it}\,dt = i\int_0^{2\pi} e^{it(k+1)}\,dt =$$

$$= i\int_0^{2\pi} \cos[t(k+1)] + i\sin[t(k+1)]\,dt = \begin{cases} i2\pi & k=-1 \\ 0 & k\in\mathbb{Z}\setminus\{-1\}\end{cases}$$

(protože prim. fce sin a cos jsou 2π periodické).