Since a thing is divisible into the parts that compose it, but the continuum cannot be composed of indivisibles, all the parts must themselves be divisible continua.[225] A continuum is, therefore, divisible into infinitely divisibles. In other words, there are no ultimate parts in a continuum. There is no bottom layer of parts into which a continuum can be divided. In this sense, a continuum is without a foundation. Every part of an n-dimensional continuum has non-zero size and is extended in n-dimensions, and every part of that continuum is itself capable of being divided into parts with non-zero size and extension in n-dimensions. Every part of a continuum has some size, but there is no smallest size.[226]

Proposition 23. *The matter of objects is divisible into ever-divisibles. It is without a foundation.*

Incidentally, we may note that this supports Aristotle's view about limits. The matter of an object does not contain limits as parts. It is not made up of limits. Surfaces exist as boundaries of bodies. The extension—the stuff, so to speak—of bodies cannot be reduced to lower-dimensional surfaces. An extended object has two ontologically different ingredients. It has parts which are divisible into ever-divisibles. And it has limits that exist only as limits of the parts. This ontological difference is, I propose, also the reason why it makes no sense for Aristotle to assume that an object is divided into its limits and the rest. A limit is not the right sort of entity to be divided from an object. An object is divided into its parts, but limits are not parts.

To be sure, this does not entail that there are no limits or that we cannot distinguish the matter of bodies from their limits. But it does entail that the standard representation of a line as a set of points is, in Aristotle's case, deeply misleading. For, if we conceive of a line as a set of points, we readily distinguish open and closed intervals according to the question whether they include their boundary points or not. But for Aristotle, if I am right, this distinction is problematic. A limit is not something that an object might have or fail to have. If lines are just sets of points, there is a straightforward sense in which a line can either have or not have a limit point. But for Aristotle a limit is not something on par with parts. A limit point exists only as a boundary of a line or of parts of a line. All parts have their limits by metaphysical necessity. In this sense, the division of an object is a division into its parts, all of which have their boundaries. There is no way to separate a boundary so that it exists on its own.

225 For the premiss that a thing is divisible into the parts that compose it see *Ph.* VI.1 231b10–11.

226 In this sense, I believe that Aristotle's continuum exhibits some similarity to what is called nowadays 'atomless gunk'. The term is found in Lewis 1991. However, a comparison is difficult because 'gunky' topologies often do away with points altogether and treat them as nested regions. Cf. Arntzenius 2011; Roeper 1997. Aristotle, on the other hand, does recognize points in his ontology, though they are dependent on higher-dimensional objects. Aristotle's conception seems to be closer to Brentano's conception of the continuum than to a point-free, region-based topology. Cf. Brentano 1976. Nevertheless, given that the matter of objects is infinitely divisible and has no ultimate parts, the parts of the matter exhibit a gunky structure. That is to say, a limit is ontologically different from a part of an object and the structure of parts can be seen as atomless gunk. For a modern formal reconstruction of the Aristotelian continuum see Roeper 2006.

7

Contact and Continuity

7.1. Introduction

In this chapter I will argue that Aristotle uses the concept of continuity to explain integral wholes.[1] Continuity is, according to Aristotle, a kind of unity. Moreover, the primary field of application of the concepts of continuity and contact are physical substances. Physical substances are, in a non-arbitrary way, unified objects. That is to say, they have a nature or some causal principle which explains their unity. In the Introduction to my work I have argued that, for example, the zoologist must be able to explain these facts about animals. In her study of animals she makes crucial use of notions such as continuity and contact.

Note that in saying that the notion of continuity has its primary application with regard to physical entities I do not want to claim that mathematical magnitudes cannot be continuous. Of course, they can. There is no problem with saying that a certain mathematical line is continuous. However, insofar as the concept of continuity is tied to the concept of a non-arbitrary unity which is explained by some causal story, its study is the task of the physicist. The mathematician does not explain the unity of objects with regard to a causal story. In this sense, the present chapter belongs to a study of bodies *qua* physical bodies.[2]

This chapter as a whole is an interpretation of basic concepts in *Physics* V.3. I have divided my discussion into two main parts. In the first part I will address Aristotle's definition of the main notions found in Section 7.2. After expounding the

1 For a modern treatment of integral wholes cf. Simons 1987; Varzi 2007.

2 It is difficult to tell what grounds the unity of mathematical objects. I think it is plausible to assume that the mathematician does to a certain extent construct her objects. Aristotle makes some hints towards such a view, though he isn't explicit about it. Cf. *de An.* III.6 and *Metaph.* IX.9. If that is the case, the unity of mathematical objects is derived because it is—to a certain extent at least—up to the mathematician what counts as *one* object. For instance, there is no fact of the matter whether the mathematician deals with two rectangles that touch or with one rectangle that has two parts. The mathematician can stipulate that. But in the case of physical objects there is an answer that does not depend on a stipulation by the physicist. There is a fact of the matter whether there is one continuous object or rather two discontinuous things that are in contact. My body (minus my arm) and my arm are continuous, and my whole body is in contact with the air that surrounds me. The explanation of this depends, as I shall argue, on facts about my nature. These facts, it seems to me, are missing in the case of mathematics. Be that as it may, in what follows I shall concentrate on Aristotle's theory of the continuous as a type of connection between physical entities. This is consistent with various views about Aristotle's theory of mathematics.

definition I will argue that the definition of contact and continuity are grounded by considerations about the ontology of physical objects (Section 7.3). The distinction between continuity and contact presupposes a certain metaphysical background. For expository reasons it is, however, convenient to first elucidate the definitions and then discuss the metaphysics behind them.

Nevertheless, the reader should be aware that my aim is to provide the conceptual tools for a study of bodies. Accordingly, I will not discuss the application of Aristotle's definitions to motion.[3] I am concerned with Aristotle's theory of extended objects and more specifically his theory of bodies. Accordingly, I will not discuss all the notions mentioned in the *Physics* V.3 in full, but only insofar as they are pertinent to the distinction between continuity and contact and the application to magnitudes.[4]

7.2. Contact and Continuity—the Formal Theory

7.2.1. The first definitions: coincidence, separation, and contact

Aristotle begins by defining the basic notions of 'coincidence', 'being separate', and 'being in contact':

> Things are said to be coincident in place when they are in the same primary place and to be separate when they are in different places. Things are said to be in contact when their extremities are together.[5] (*Ph.* V.3 226b21–23)

Aristotle's use of the word 'I say (*λέγω*)' together with the identification of two terms suggests that he intends for these lines to state the definitions of the terms involved.[6] In what follows I use the term 'coincidence' for spatial coincidence, that is, being together

[3] The context of *Physics* V.3 shows that the application to a theory of motion is highly relevant. For the fifth and sixth book of the *Physics* are concerned with motion and are even considered to be part of an independent treatise 'On Motion (*Περὶ κινήσεως*)' which either contains Books V–VIII or Books VI–VIII of the *Physics*. The most recent discussion is Odzuck 2014, 15–24.

[4] Before I begin my interpretation of the chapter it should be noted that there is a textual problem. Ross transposes lines 227a7–9 and 226b26–27. See Ross 1936, 626–8. I think the transposition is unnecessary, but since for our purposes nothing hangs on it, I will not address the transposition. The transposition affects Aristotle's definition of 'between', which plays little role in my analysis. It is a notion that is most explicitly linked to the analysis of change: 'That which a changing thing, if it changes continuously in a natural manner, naturally reaches before it reaches that to which it changes last, is between' (*Ph.* V.3 226b23–26/227a7–10). For a discussion of the definition see Dehn 1975; Solmsen 1960; Waschkies 1977, §13.

[5] *ἅμα μὲν οὖν λέγω ταῦτ' εἶναι κατὰ τόπον, ὅσα ἐν ἑνὶ τόπῳ ἐστὶ πρώτῳ, χωρὶς δὲ ὅσα ἐν ἑτέρῳ, ἅπτεσθαι δὲ ὧν τὰ ἄκρα ἅμα.*

[6] Of course, *λέγω* may also be used to signal that the definition serves the present purposes, but may be refined, or even revised, later. But the present context seems to indicate something stronger. Especially since in the treatment of contact in *GC* I.6, which relies on the definitions stated in the *Physics*, Aristotle explicitly refers back to the definition in the *Physics* with the phrase 'it has been defined' (*GC* I.6 323a2). Later in the chapter, he expands the definition and again calls it explicitly a 'definition' (*GC* I.6 323a22). The main difference between *Ph.* V.3 and *GC* I. is that in *GC* I.6 Aristotle presents a more demanding conception of contact, which includes that the things in contact can act upon one another. However, if my treatment of *Ph.* V.3 is correct, this is a natural extension and, to a certain extent, presupposed in *Physics* V.3.

(*ἅμα*) in place. For material coincidence, that is, a sharing of parts, I use the term 'overlap'.

7.2.1.1. COINCIDENCE AND SEPARATION

I suggest that Aristotle's definitions be stated in the following way:

> **Proposition 24.** *Things are coincident if and only if they are in the same primary place.*

> **Proposition 25.** *Things are separate if and only if they are in different primary places.*

Two questions immediately arise: how should 'being separate' and, accordingly, 'being coincident' be understood? What is the connection of this definition to the definition of primary places in *Physics* IV.1–5? I will start with the first question. Being separate could either mean that the places of the object are not fully the same, or it could mean that the places of the objects are disjoint. Although Aristotle does not discuss these options, I assume that Proposition 25 means that the objects are disjoint. Their primary places do not overlap at all. My assumption is based on systematic considerations. Suppose, for example, a smaller object is fully contained by a larger object (see Fig. 7.1).

If being separate in place meant that the objects occupy merely distinct, but not necessarily disjoint, places, A and B would be separate because their primary places are distinct. Yet, I believe that it is more plausible to say that A and B are in fact coincident. After all, the place of B lies wholly within the place of A. In this case, I think that it is wrong to say that A and B occupy separate places. In other words, if being separate in place meant that the objects occupy merely distinct, but not necessarily disjoint, places, we could not make a distinction between the case of two partially coinciding bodies, that is, bodies that have some, but not all of their parts in the same place, and disjoint bodies, which have no parts in the same place.[7] Both are, according

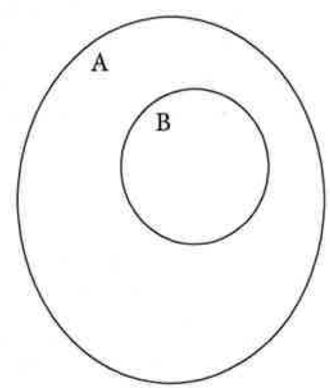

Fig. 7.1.

[7] In light of *Physics* IV.1–5 the ascription of a place to parts might seem problematic. I discuss this below.

to this interpretation, separate. But this seems wrong. I believe that the distinction between partially coinciding bodies and disjoint bodies is crucial to Aristotle.[8]

In the light of these difficulties, I think we have good (philosophical) reasons to assume that separation means having disjoint places. Therefore, I assume that Aristotle would accept the following:

Proposition 25.* *Things are separate if and only if none of their parts is in the same primary place.*

In this sense, Proposition 25* can be graphically represented as two disjoint sets (see Fig. 7.2):

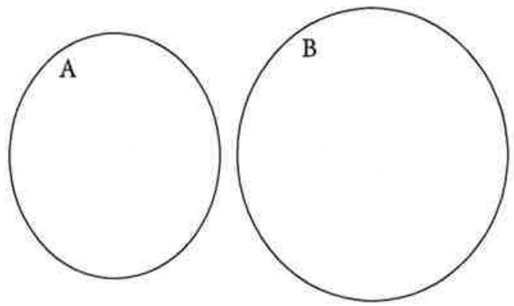

Fig. 7.2.

A and B are separate because they have no parts in the same primary place. Whatever part of A and whatever part of B one takes, the parts are always in disjoint places. This, however, raises the question how we should understand the definition of coincidence. Due to my assumption that the places of separate things are disjoint (rather than just different), I interpret Aristotle's definition of spatial separation in a strong way. Being in separate places is more than being in different places. It means to be in disjoint places. What then does 'coincidence' mean? Are being separate and being coincident contraries or contradictories? Are these definitions meant to deliver an exhaustive and exclusive classification? Do things coincide if and only if *all* of their parts are in the same primary place? In this case, coincidence would be the contrary of being separate. Or are things coincident if and only if *some* of their parts

[8] Of course, we might distinguish between partial separation, i.e. not having all parts in the same place, and full separation, i.e. having no parts in the same place. But it seems to me that partial separation should rather be described as a case of coincidence. The crucial difference is between things that have *no* parts in the same place and things that have *some* parts in the same place, not between things that have *some* parts in the same place and things that have *all* parts in the same place. Moreover, to take partial separation as basic would needlessly complicate things. For we would have to distinguish three relations which cannot obtain at the same time, i.e. full and partial separation and coincidence. Whereas if coincidence is defined as having *some* parts in the same place, as we will do, this is compatible with having *all* the parts in the same place. Thus, we need only define separation and coincidence.

are in the same primary place? In that case, coincidence would be the contradictory of separation. I propose to choose the second option. That is to say, in Proposition 24 Aristotle means to say that things are coincident if and only if *some* of their parts are in the same place. A and B are coincident if and only if there is a part of A and a part of B in the same place, but not necessarily *all* the parts of one or both objects.

Though it is a little harder on the Greek, I think we can adduce a systematic reason that supports my reading. First, it is preferable from a systematic point of view that the definitions be contradictories. There should be no middle ground between coincidence and separation. Suppose that the left half of one object and the right half of another object are in the same place. It is natural to say that the objects coincide. Objects coincide if *at least one* part of each is in the same primary place. But if we were to assume that coincidence means having *all* parts in the same place and separation means being in disjoint places, we could not describe this situation because the two objects would neither be coincident nor separate.[9]

Or, to return to the example of the object which lies wholly within the extension of a larger object: these objects would not be coincident because their respective primary places would be different. Or consider the notion of contact: If contact is defined as having the boundaries in the same spot, two cubes that touch at just one side aren't in contact after all, because Aristotle would have defined it as having complete overlap of boundaries.[10]

In other words, I assume that *partial* coincidence counts as coincidence. It is not an option to maintain that the definitions are not exclusive and things can be both coincident and separate. This seems to me to go against the tone and purpose of the classifications. Coincidence is—in the terms of the mereological calculus—overlap of the places. Two things overlap if they have a part in common. And two things coincide if they have some parts in the same primary place (see Fig. 7.3).

A and B coincide because part of A and part of B are in the same primary place. Therefore, I shall assume the following:

Proposition 24.* *Things are coincident if and only if they have some parts that are in the same primary place.*

Coincidence implies that some parts of the entities in question are fully coincident. Full coincidence can therefore be defined as coincidence of all the parts. A and B are fully coincident if and only if they have all their parts in the same place. In what follows I will assume that Propositions 24 and 25 should be understood as the reformulated starred versions.

[9] Of course, we could modify the definition of separation, but, as I said, I believe that this is an unattractive position.

[10] One might argue that in the case of the cubes there are really six independent extremes, and only one of these extremes of each cubes coincides fully. I would answer that the problem would reappear if the cubes were arranged in such a way that only part of the side is in contact. Moreover, as I have argued in Section 6.3.1 Aristotle's definition of boundary in *Metaphysics* V.17 is a definition of the entire boundary.

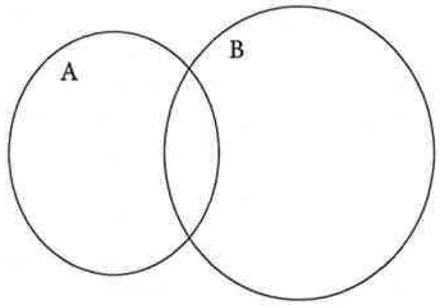

Fig. 7.3.

Let me now briefly discuss the connection to *Physics* IV.1–5. Here my interpretation differs considerably from the interpretation by Alexander. He believes that the definition of coincidence refers to the parts of a thing that are 'in the same primary place'.[11] According to this interpretation my arm and my leg are coincident (*hama*) because they are in the same primary place. Having the same primary place does not mean here that my arm and my leg occupy exactly the same region, but that the place of my arm and my leg is the same because they have the same place as I as a whole have. The background for Alexander's definition is Aristotle's account of place in *Physics* IV.1–5. In *Physics* IV.1–5 Aristotle claims that the parts of a thing have the same place as the whole.[12] In this sense, a sphere, the left half of the sphere and the right half of the sphere have the same place and, according to Alexander, coincide.

I think that this interpretation is not tenable. First, whenever Aristotle speaks elsewhere about coincidence or two bodies being in the same place, he has in mind the case in which the places of two or more bodies have some of their *parts* in the same place.[13] The places of the parts differ from each other and a part of a whole does not have the same place as the whole. Moreover, if we understand the definition in Alexander's sense we create the additional difficulty of how one should understand the notion of contact. For in *Ph.* V.3 226b21–23, quoted above, Aristotle unambiguously says that things in contact have their extremities in the same primary place.[14] This is not compatible with the theory as presented in *Physics* IV.1–5, where boundaries do not have a place in the strict sense at all. Alexander is aware of this problem and argues that Aristotle does not employ 'place' in the same way in his account of coincidence and contact. Rather, in the first definition Aristotle uses the strict notion of place, whereas in the second he uses the mathematical notion of *ἐφαρμόζειν*.[15] The obvious

[11] Cf. Simp. *in Ph.* 868.25ff. [12] *Ph.* IV.4 211b25–26.

[13] *de An.* II.7 418b17; *de An.* I.5 409b2–3; *Metaph.* III.2 998a18–19; *Ph.* IV.5 212b25. Notice that the question is not whether bodies can in fact coincide. Aristotle denies that in these passages. The question rather is how the definitions have to be understood and what *would* count as coincidence, even if, as a matter of fact, coincidence is impossible.

[14] See my discussion in Sections 7.2.1.2 and 7.3.1. [15] Cf. Simp. *in Ph.* 870. 24–26.

problem with this strategy is that the definition of contact and the definition of being in the same place are not related. The term 'coincident (*ἅμα*) is used homonymously. That is to say, the the term 'coincidence' in the statement 'Things are coincident if they have parts in the same primary place' and in the statement 'Things are in contact if their boundaries are coincident' is used homonymously. This, I propose, is hard to square with the fact that these definitions seem to build upon each other (see next section).

Thus, I think that Aristotle's definition of coincidence should be understood as overlap of places or as parts having the same place. According to this interpretation, the left half of a sphere and the right half of a sphere have separate primary places and, therefore, do not coincide. No part of the left half is in the same place as a part of the right half. Accordingly, I believe that Aristotle does not use his strict notion of place that he developed in *Physics* IV.1–5. In this sense, I think that Alexander is correct in his assumption that Aristotle does not use his strict notion of place in his account of contact. But, in contrast to Alexander, I believe that this is true of Aristotle's account of coincidence in general. That is to say, Aristotle employs the same account of coincidence in his definition of contact and his definition of being in the same primary place in general.

What, then, is this sense that is employed? For one, as the example of the sphere shows the parts of an object have their own places, and simply the same place as the whole. This suggests that the primary place of something, be it a part or a whole, is the exact *spot* or *region* it occupies. In other words, the notion of primary place in *Physics* V.3 is generally close to the notion *ἐφαρμόζειν*, as Alexander suggests with respect to boundaries. A good example of the use of *ἐφαρμόζειν* that Alexander probably had in mind can be found in Proposition 24 of the third book of Euclid's elements. The proposition states that similar segments of circles on equal, straight lines are equal to one another. The argument begins with the following argument:

> For let AEB and CFD be similar segments of circles on the equal straight-lines AB and CD (respectively). I say that segment AEB is equal to segment CFD. For if the segment AEB is applied to the segment CFD, and point A is placed on (point) C, and the straight-line AB on CD, then point B will also coincide with point D, on account of AB being equal to CD. And if AB coincides with CD then the segment AEB will also coincide with CFD.[16] (Euc. III.24)

The proof starts by bringing lines AB and CD into coincidence by placing point A on point C. Since the lines are of equal length points B and D will also coincide and, hence, lines AB and CD. The lines are lying directly on each other. This, I think, is what Aristotle means when he says that two things are in the same primary place. In this sense, all things having an extension (including points), be they parts or wholes, have

[16] *῎Εστωσαν γὰρ ἐπὶ ἴσων εὐθειῶν τῶν AB, ΓΔ ὅμοια τμήματα κύκλων τὰ AEB, ΓΖΔ· λέγω, ὅτι ἴσον ἐστὶ τὸ AEB τμῆμα τῷ ΓΖΔ τμήματι. ᾿Εφαρμοζομένου γὰρ τοῦ AEB τμήματος ἐπὶ τὸ ΓΖΔ καὶ τιθεμένου τοῦ μὲν A σημείου ἐπὶ τὸ Γ τῆς δὲ AB εὐθείας ἐπὶ τὴν ΓΔ, ἐφαρμόσει καὶ τὸ B σημεῖον ἐπὶ τὸ Δ σημεῖον διὰ τὸ ἴσην εἶναι τὴν AB τῇ ΓΔ· τῆς δὲ AB ἐπὶ τὴν ΓΔ ἐφαρμοσάσης ἐφαρμόσει καὶ τὸ AEB τμῆμα ἐπὶ τὸ ΓΖΔ.*

a primary place and can coincide.[17] As we will see in the next section, even boundaries have a place, according to this theory.[18]

One might agree that a different notion of place is employed in *Physics* V.3 and IV.1–5, but still dispute whether we should speak, as I have done, in the former case of *primary* place.[19] First, in speaking of primary places I was following Aristotle's own use in *Physics* V.3 226b21–23.[20] Second, there is good reason for it because the notion of primacy is the same in *Physics* V.3 and IV.1–5. In *Physics* IV.2 Aristotle defines the primary place of an object as the place where the object is precisely located:

> There is, on the one hand, the common place in which all bodies are, and, on the other, the proper place, in which as a primary thing each body is. I mean, for instance, that you are now in the world because you are in the air and it is in the world; and you are in the air because you are on the earth; and similarly on the earth because you are in this place which contains no more than you.[21] (*Ph.* IV.2 209a32–b1)

Thus, the primary place of an object contains just the object and nothing more. In this sense, a primary place is what we could call the *proper* place of an object.[22] But surely, we need this notion of primacy to define coincidence. Two things do not coincide because they are in the same room and have the same common place. Parts of one object must be in the *exact* same spot as parts of the other object. They must have parts in the same primary or proper place.

7.2.1.2. CONTACT

The next definition Aristotle introduces is the definition of contact:

> Things are said to be together (ἅμα) in place when they are in the same primary place and to be separate when they are in different places. Things are said to be in contact when their extremities are together.[23] (*Ph.* V.3 226b21–23)

The definition of contact extends the definitions of coincidence and separation. Things are in contact if their extremities, as opposed to their (proper) parts, coincide. The intuition behind contact is that some things are as close as possible without interpenetrating each other. This implies that their boundaries must be together.[24]

[17] It should be noted that this does not imply that 'place' in this sense is ontologically independent. Rather, 'place' can be seen as an abstraction from the extension of objects. Instead of saying 'Two things are in the same place' we could say 'Part of the extension of one object overlaps with part of the extension of the other object'.

[18] The philosophical rationale behind Aristotle's definitions will be the topic of Section 7.3.

[19] This issue was raised by an anonymous commentator of OUP.

[20] Quoted above on p. 148 and in the next section.

[21] *καὶ τόπος ὁ μὲν κοινός, ἐν ᾧ ἅπαντα τὰ σώματά ἐστιν, ὁ δ' ἴδιος, ἐν ᾧ πρώτῳ (λέγω δὲ οἷον σὺ νῦν ἐν τῷ οὐρανῷ ὅτι ἐν τῷ ἀέρι οὗτος δ' ἐν τῷ οὐρανῷ, καὶ ἐν τῷ ἀέρι δὲ ὅτι ἐν τῇ γῇ, ὁμοίως δὲ καὶ ἐν ταύτῃ ὅτι ἐν τῷδε τῷ τόπῳ, ὃς περιέχει οὐδὲν πλέον ἢ σέ).*

[22] On this point see the discussion by Morison 2002, 55–66.

[23] *ἅμα μὲν οὖν λέγω ταῦτ' εἶναι κατὰ τόπον, ὅσα ἐν ἑνὶ τόπῳ ἐστὶ πρώτῳ, χωρὶς δὲ ὅσα ἐν ἑτέρῳ, ἅπτεσθαι δὲ ὧν τὰ ἄκρα ἅμα.*

[24] For the argument see Section 7.3.1.1.

Since coincidence or togetherness (ἅμα) is spelled out in terms of being in the same primary place, we can say:

> **Proposition 26.** *Things are in contact if and only if parts of their extremities are in the same primary place.*[25]

It may sound, at first, not very intuitive to say that extremities have a (primary) place. However, as I have argued, the notion of place used here is not to be confused with the notion employed in *Physics* IV.1–5. The notion of primary place here is the notion of the *exact spot* an entity occupies. Since Aristotle is a realist concerning boundaries, it makes sense to say that the two-dimensional boundary of a three-dimensional object has a two-dimensional place. The claim is that, if two bodies are in contact, their extremities are in exactly the same two-dimensional spot. Moreover, I assume that the sense of coincidence in this proposition is subject to the same modifications I have argued for in regard to the definition of coincidence in Proposition 24. That is to say, things are in contact if *parts* of their boundaries coincide. Graphically, this can be represented as follows:

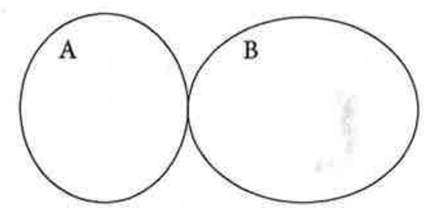

Fig. 7.4.

The objects A and B are in contact because a point on the periphery of A and a point on the periphery of B are in the same place (see Fig. 7.4). In this sense, A and B are connected. In specifying the region or place that the boundaries of A and B occupy, a part of the whole region is occupied by parts of both boundaries. If this is correct, Aristotle's account of contact shares an important feature with modern conceptions of contact. One can go from A to B without ever going through the exterior of A and B.[26]

[25] Strictly speaking, if two spheres touch at a point, they do not have a *part* of their boundary in the same place, because points are not *parts* of the surfaces which are the boundaries of the two spheres. See Section 6.3.4.3. Strictly speaking, we should say that things are in contact if and only if parts of their boundaries or parts of the boundary items of the boundary are in the same place. In other words, two bodies are in contact if and only if their boundaries are in the same two-, one- or zero-dimensional place.

However, to keep things simple I will ignore that difficulty here. There is, as I have said, also the additional question whether the parts of an object have places at all.

[26] Cf. Casati and Varzi 1999, 80: 'In mereotopological terms, the difference is that in the first case (contiguity), one can go from any one part of one sphere to any part of the other without ever *going through the exterior* of the whole: this is what makes them connected. In the second case (continuity), one can go from any part of one half to any part of the other half without ever *leaving the interior*.' As I will show later, Aristotle's conception of continuity also fits this pattern.

It is, however, crucial to note that A and B still remain separate. It is wrong to say that they coincide. They do not coincide, because they do not have *parts* in the same place. Their *boundaries* have parts in the same place, but not the objects themselves. Aristotle makes this point explicit in *On Generation and Corruption*.

Assuming, therefore, that 'to touch' is—as we have defined it in a previous work—'to have the extremities together', only those things will touch one another which, being separate[27] magnitudes and possessing position, have their extremities 'together'.[28] (*GC* I.6 323a3–6)

Hence, Aristotle seems to accept the following.

Proposition 27. *Things are in contact if and only if all of their parts are in disjoint primary places and parts of their extremities are in the same primary place.*

Proposition 27 spells out the important feature of Aristotle's definition: objects can be in different primary places though their limits are in the same primary place. Things are separate if and only if they occupy disjoint primary places. Things are coincident if either they or their parts are in the same primary place. And things are in contact if their boundaries are in the same place.

At this point the distinction between parts and boundaries is crucial.[29] If boundaries were parts, there would be a contradiction, because Proposition 27 would imply that things touch if none of their parts is in the same primary place and some of their parts are in the same primary place. We avoid the contradiction by explicitly distinguishing between boundaries and parts of an object.

Boundaries have at least one dimension less than (real) parts. Hence, boundaries can be in the same place without the bodies to which they belong being in the same place. The place of a body has three dimensions. If two bodies coincide, they have some of their parts located in the same three-dimensional region. But boundaries of bodies can be in the same place without the bodies that are bounded by them being in the same place. The place of the surface of a body is two-dimensional. But the places of the parts of a body are three-dimensional.

Thus, if the boundaries of two bodies coincide, they are in the same two-dimensional place. The bodies themselves, however, do not coincide, since their respective places are disjoint. There are no *parts* of the bodies that are in the same place. Therefore, corollary 27 should be read as:

[27] I follow the suggestion by Joachim to read '*διηρημένα*' instead of the manuscripts' '*διωρισμένα*'. Cf. Joachim 1926, 144. 'Separate' has another meaning, too, which I will discuss later. For it implies that the magnitudes in question are not only spatially separate, but also ontologically separate, i.e. they are two independently existing objects.

[28] *Εἰ οὖν ἐστίν, ὥσπερ διωρίσθη πρότερον, τὸ ἅπτεσθαι τὸ τὰ ἔσχατα ἔχειν ἅμα, ταῦτα ἂν ἅπτοιτο ἀλλήλων ὅσα διηρημένα μεγέθη καὶ θέσιν ἔχοντα ἅμα ἔχει τὰ ἔσχατα.*

[29] See Section 6.3.4.3.

Proposition 27.* *Things are in contact if and only if none of their (composing and same-dimensional) parts are in the same primary place and parts of their lower-dimensional boundaries are in the same primary place.*

Since things are located where their parts are located, there is no contradiction. This may provoke the following question: If an object is located where its parts are located, is it also located where its boundary is? Of course it is. For, the boundaries, as Aristotle saw, do not increase the size of the objects. The parts of an object, we may say, occupy a topologically open place. The parts together with the limit occupy a topologically closed place. There is no contradiction between the claim that the limits of two objects are coincident and yet the objects separate, nor does an object taken with its limit occupy a larger place than the same object taken without its limit.

7.2.2. The second definitions: in succession, contiguity, and continuity

The Propositions 24, 25, and 26 introduce fundamental positional relations that hold between extended objects. The following definitions extend the first definitions in two respects: first, they introduce further structural considerations. Proposition 26, for example, is used explicitly in the definition of contiguity. In what follows I will state the definitions and clarify the notions involved. In the second part of this chapter I will then argue that the definitions of continuity and contiguity presuppose a certain metaphysical background.

7.2.2.1. BEING IN SUCCESSION

Since I have already discussed the notion of being in succession or next to in Section 5.2.1 and this notion will play only a minor role in what follows, I will only briefly recapitulate it.[30]

A thing is in succession (*ἐφεξῆς*) when it is after the beginning in position or in form[31] or in some other respect in which it is definitely so regarded, and when further there is nothing of the same kind as itself between it and that to which it is in succession, e.g. a line or lines if it is a line, a unit or units if it is a unit, a house if it is a house (there is nothing to prevent something of a different kind being between).[32] (*Ph.* V.3 226b34–227a4)

Aristotle describes an ordering relation which is based on notions of 'after' or 'prior' and 'posterior' and the notion of 'between'. An example are the natural numbers:

$\bullet^1 \quad \bullet^2 \quad \bullet^3 \quad \bullet^4 \quad \bullet^5 \quad \bullet^6 \quad \bullet^7 \quad \bullet^8 \quad \bullet^9$

The number one is the beginning and all the other units are ordered according to their numerical value. All the other numbers come after the one. The three is in

[30] See also Section 6.4.5.2.

[31] The manuscript F reads *φύσει*, which might be preferable. See Waschkies 1977, 175.

[32] *ἐφεξῆς δὲ οὗ μετὰ τὴν ἀρχὴν ὄντος ἢ θέσει ἢ εἴδει ἢ ἄλλῳ τινὶ οὕτως ἀφορισθέντος μηδὲν μεταξύ ἐστι τῶν ἐν ταὐτῷ γένει καὶ οὗ ἐφεξῆς ἐστιν (λέγω δ' οἷον γραμμὴ γραμμῆς ἢ γραμμαί, ἢ μονάδος μονὰς ἢ μονάδες, ἢ οἰκίας οἰκία· ἄλλο δ' οὐδὲν κωλύει μεταξὺ εἶναι).*

succession to the two because there is no other number between. Similarly, the four is in succession to the three and the five is in succession to the four. But the five is not in succession to the three because there is something of the same kind between them, namely the four. A slight complication should be noted. Aristotle suggests that being in succession is not symmetric. Thus, the two is next to the one, but the one is not next to the two.[33] However, since we are mainly concerned with the distinction between continuity and contact, which is symmetric, I will ignore this distinction. It mainly matters when it comes to the ordering of time stretches or motions. Therefore, I will define the symmetric case and treat ἐφεξῆς as if it meant next to.

Proposition 28. *x is next to y if and only if [1] x and y are members of a specified order and [2] no other member of the order is between x and y.*

7.2.2.2. CONTIGUITY

The next concept Aristotle introduces is the concept of contiguity. The definition of contiguity is an extension of the previous definition. The difference lies in the postulation that in addition to being in succession the entities in question are touching.

That which, being in succession to / next to something (ἐφεξῆς), is touching it, is contiguous.[34]
(*Ph.* V.3 227a6)

One immediate consequence of the definition is that only objects that are in a place can be said to be contiguous.[35] This is a consequence of Proposition 26, which says that things are in contact if and only if their boundaries are in the same place. Therefore, contiguity introduces not only a new structural feature, but one that limits the area of application. Numbers[36] and parts of the soul[37] can be in succession or next to each other, but not contiguous. The definition of contiguity is applicable only to extended things.[38]

The role of extension is not an accidental feature of Aristotle's definition. Rather Aristotle uses the definition of contiguity to draw a contrast between points (στιγμαί) and monads (μονάδες):

Hence, if as some say points and units have a separate existence of their own, it is impossible for the two to be the same; for points can touch while units can only be in succession.[39]
(*Ph.* V.3 227a27–30)

[33] *Ph.* V.3 227a5. [34] *ἐχόμενον δὲ ὃ ἂν ἐφεξῆς ὂν ἅπτηται.*
[35] 'Place' is here understood in the looser sense I explained above.
[36] Cf. *Ph.* V.3 227a20.
[37] Cf. *De An.* II.3 414b20–415a1.
[38] Or to things that are 'closely connected' to extended things, like points. Of course, this does not necessarily mean that the objects must be *spatially* extended. Time stretches or motions can be contiguous, though they are not spatially extended. But it is clear that spatially extended objects are paradigmatic cases for things that are contiguous.
[39] *ὥστ' εἰ ἔστι στιγμὴ καὶ μονὰς οἵας λέγουσι κεχωρισμένας, οὐχ οἷόν τε εἶναι μονάδα καὶ στιγμὴν τὸ αὐτό· ταῖς μὲν γὰρ ὑπάρχει τὸ ἅπτεσθαι, ταῖς δὲ μονάσιν τὸ ἐφεξῆς.*

Aristotle argues that a point and a monad (assuming that they exist separately) cannot be the same.[40] Points belong to a theory of contact, but monads do not.[41] Hence, a point and a monad must be different. This has, I think, some interesting ramifications for Aristotle's method here. Aristotle does not first specify a domain of objects and then postulate various relations among those objects. Rather, the relations he specifies are such that they contain implicit information about the type of objects that may stand in these relations. If objects are in contact, they are extended. If something is not extended, it cannot be in contact with something else.

Proposition 29. *x is contiguous with y if and only if [1] x is next to y and [2] x and y are in contact.*

7.2.2.3. CONTINUITY

The definition of continuity is the last definition Aristotle gives.

The continuous is that which is contiguous to something [or: is something contiguous], but I call them continuous only when the limits at which they are touching become one and the same, and, as the name signifies, hold together.[42] (*Ph.* V.3 227a10–12)

Aristotle introduces the continuous as something contiguous.[43] But the continuous differs from the contiguous in the following respect: (1) the boundaries, Aristotle says, become one and the same, and (2) the boundaries hold together.

Proposition 30. *x is continuous to y if and only if [1] x and y are contiguous and [2a] the boundaries at which they are in contact become one and the same and [2b] the boundaries hold together.*

In what follows I shall discuss the conditions [2a] and [2b] and contrast them with Aristotle's concept of contact. Or, more precisely, I will ignore the fact that according to these definitions only things that are in succession or next to each other can be contiguous or continuous. The reason is that our main topic is the distinction between continuity and contact as two ways of being connected. And this distinction can be explained without any specific ordering relation. Moreover, as I have used the symmetric notion of next to in order to explain ἐφεξῆς, there is little at stake in any case. For if two things are in contact, it is implied that they are next to each other.

[40] Aristotle is thinking of a Platonic or Pythagorean theory. Such a theory assumes that points and monads are self-subsistent. Aristotle's remark about a Platonic theory in this chapter may have a historical explanation. For, it is argued by Waschkies 1977 §15ff. that Aristotle presents here an early theory of the continuum which is in the tradition of Platonic theories as we find them in the *Parmenides*.
[41] Notice that points themselves cannot be in contact, since they have no limits. But they are part of an explanation of how lines are in contact. Lines are in contact if and only if their end points coincide. Or, if we use the looser sense of contact, points are in contact by coinciding as wholes. For the distinction between the looser and the stricter sense of contact see Section 6.4.5.2.
[42] *τὸ δὲ συνεχὲς ἔστι μὲν ὅπερ ἐχόμενόν τι, λέγω δ' εἶναι συνεχὲς ὅταν ταὐτὸ γένηται καὶ ἓν τὸ ἑκατέρου πέρας οἷς ἅπτονται, καὶ ὥσπερ σημαίνει τοὔνομα, συνέχηται.*
[43] Note the word ὅπερ, which Aristotle often uses to express a statement of definition.

The basic thought is that in the case of continuity the boundary between X and Y is one. In this sense, if one goes from X to Y one does not come across two boundaries where X and Y touch, but only one boundary. One might think, in the case of a continuous entity, it does not make sense to speak of a boundary between the parts at all. This seems to have been the position of Alexander who is quoted by Simplicius as saying that 'whereas in the case of things that are continuous, even the one is destroyed; for things are continuous when in actuality there is no boundary in between' (Simp. *in Ph.*, 570.6–7). If there is a boundary between the parts, how can the object be continuous? I will discuss the status of internal boundaries in detail in Section 7.3.2, but let me note here that such an objection may, at least partly, be based on a misunderstanding of what a boundary between two parts is. A boundary does not necessarily separate things. Nor does it mark off an articulation in an object. In fact, as the quote above suggests, an internal boundary rather has the opposite role of unifying things because the parts of an object 'hang together' at a boundary. Of course, this metaphor should not be stretched too far, either. But it aptly describes the *topological* role internal boundaries have. Parts of continuous objects are connected in such a way that in going through the object you never leave the interior of the object. If you go from the left half to right half of a sphere, one never leaves the interior of the sphere, but, of course, one leaves the left half. In saying that the halves are connected at a boundary, one says, in the first instance, only that the left half ends exactly where the right half begins. On this account, it does not make sense to say that there is no boundary between the parts. This would imply that the things are not continuous, that is, topologically unified. The connection, however, does not mean that the boundary is literally 'doing' anything, be it separating or holding together. The question, therefore, is not whether there is one or no boundary between the parts of a continuous object. The real question is how we should conceive of this boundary in various different cases.

Continuity is, therefore, a relation between the parts of a single object. Thus, if X and Y are continuous, they are parts of one object. In contrast to that, contact is a relation between two independent objects, both having their own boundaries. My claim is that the difference between continuity and contiguity cannot—in its primary application—be explained without an appeal to the ontology of objects that stand in the relation. The explanation of the difference between contiguity and continuity is based on metaphysical and physical facts about the objects standing in this relation. This is the main claim of the next section.

7.3. Contact and Continuity—the Metaphysical Theory

The difference between continuity and contiguity is based on a difference in the way the entities are connected. The boundaries, as Aristotle says, become one. This, however, is explicable by focusing on the notion of contact alone without taking into consideration the special ordering relation that contiguity implies. In this sense, I will

treat contact and contiguity on a par. This is also consistent with Aristotle's use of these terms in other passages, where he contrasts contact and continuity, rather than contiguity and continuity.[44]

7.3.1. *The possibility of contact*

Contact is defined as the coincidence of the respective boundaries. There are two boundaries in the same place (Proposition 26). This definition is, I believe, intriguing. What led Aristotle to this definition? Of course, one may say that Aristotle stipulated this definition, end of story. But this does not release us from the philosophical task of explaining Aristotle's definition and elucidating why one should accept that definition within his system. In what follows I shall argue that there is an elaborate philosophical view behind the definition. We can explain why Aristotle stated the definition in precisely this way. Let us therefore first review Aristotle's definition of contact:

> **Proposition 26.** *Things are in contact if and only if parts of their extremities are in the same primary place.*

In considering the philosophical importance and relevance of Aristotle's views on the relation of contact it may help to ask ourselves what constraints are set on *any* theory of contact. I propose that every theory about the relation of contact has to do justice to two intuitions. First, objects that are in contact occupy disjoint or separate places. Second, objects that are in contact are as close as possible. That is to say, if certain objects are in contact it is not possible for other objects to be closer than the objects in contact.[45] Aristotle's definition, as I have shown, does justice to these intuitions. But I think we can conclude something stronger as well. Not only does Aristotle's definition do justice to these two constraints, but, in the confines of his theory, this is in fact the *only* way to satisfy these two constraints. In the next section I will show why that is the case.

7.3.1.1. WHY THE BOUNDARIES HAVE TO BE COINCIDENT

In order to show that the boundaries have to be coincident, let us introduce an alternative way of characterizing contact which is based on the intuitions mentioned above:

> **Contact.*** *Two things x and y are in contact* if and only if (a) both x and y have a place, (b) no (spatial) entity z that is extended in equal or more dimensions is between them and (c) x and y do not coincide.*

[44] Cf. *Metaph.* V.4 1014b22–26, *Ph.* IV.4 211a29–b5. In this vein Ross comments on *Ph.* V.3: 'In no other passage other than the present (and the corresponding passage in *Metaph.* K) is there any attempt to distinguish *ἐχόμενον* from *ἁπτόμενον*' (Ross 1936, 627).

[45] On these constraints see also Zimmermann: 'An initial constraint on any adequate theory of the nature of contact is this: it should not imply that some pairs of extended objects which are in contact are closer to one another than other such pairs. The notion of contact is essentially that of a limit on how close things can get without either interpenetration or the sharing of parts' (Zimmermann 1996, 9).