3

Body in the Context of Physical Science

In the following two chapters I will defend the idea that the study of bodies should be seen as a part of the conceptual underpinnings of physical science and will relate it to the study of magnitudes as it occurs in mathematics. Obviously, this requires some more general comments on Aristotle's conception of physical science vis-à-vis his conception of mathematics. To spell out the details of this would require a study of its own, which I will not attempt here. Instead, I will restrict my discussion to the defence of four claims. The first two claims comprise Chapter 3, the third and fourth comprise Chapter 4.

1. A study of bodies and magnitudes is central to the framework of Aristotle's conception of physical science (Section 3.1).
2. This study is not a mathematical investigation. It is a study of body and magnitudes insofar as they are the bodies and magnitudes of physical substances (Section 3.3).
3. The mathematician studies body and magnitudes, too. But she is engaged in a different field of scientific endeavour because she studies magnitudes not insofar as they are physical. This study involves a separation from motion (Section 4.1).
4. Though physical science is not a branch of mathematics, the physicist can draw on mathematical theorems (Section 4.2).

Very briefly, the structure of my overall argument that connects these claims is the following: in order to do physical science or, more specifically, a science of motion, one has to include a study of bodies. The reason is that many notions discussed by Aristotle in his physical works straightforwardly include reference to bodies and related notions such as contact, continuity, or boundary (for example the discussion of place, the infinite, or action and passion). The question, however, which immediately arises is in what way this study differs from a mathematical study. The answer is that the point of view of the former study is inextricably linked to the physical nature of the substances to which body and magnitudes belong. It considers the ontology of body insofar as it is the body of a physical substance. A physical substance has a body and it is the task of the physicist to consider the nature of body insofar as it belongs to a physical substance. In this sense, it is a study of physical bodies.

What, then, does the mathematician study? She studies bodies as if they were separate. That is, in mathematics one does not take into account that body and magnitudes belong, from the standpoint of ontology, to physical substances. Since the mathematician and the physicist differ in the focus of their science, they are different scientific fields. Yet, these fields are not on a par when it comes to discovering the real nature of body and magnitude. Since body and magnitude ontologically depend on physical substances, the physicist, taking this into account, investigates the true nature of body and magnitude.

3.1. The Importance of an Account of Body for Physical Science

In this section I argue that the study of body is indispensable for physical science because it is part of the conceptual underpinnings or principles of the particular branches of physical science. In this sense, a study of body should be connected to *Physics* III–IV where Aristotle elucidates and systematizes basic concepts that are crucial to physical science. In Section 3.3 I will further argue for the distinctively physical character of this study.

My main claim is that the physicist studies what belongs to animals insofar as animals are three-dimensional and connected entities. In the course of her study of animals the physicist has to rely on concepts such as continuity, contact, being extended, having a boundary, and so on. That is to say, she not only studies animals as such, but also what belongs to animals insofar as they have the aforementioned features. It is impossible to give a satisfactory theory of animals without those concepts and, hence, the success of the physical sciences crucially hangs on a grasp and elucidation of these concepts. Consider the following passage from the *Parts of Animals*:

> There is a resemblance between the osseous and the vascular systems; for each of them has a central part in which it begins, and each forms a continuous whole. For no bone in the body exists as a separate thing in itself, but each is either a portion of what may be considered a continuous whole, or at any rate is linked with the rest by contact and by attachments; so that nature may use adjoining bones either as though they were actually continuous and formed a single bone, or, for purposes of flexure, as though they were two and distinct.[1]
>
> (*PA* II.9 654a32–b2)

Aristotle compares osseous and vascular systems. Both have central parts and form continuous wholes. What does this mean? What does the notion of continuity express here? It seems to be a kind of connection of the parts, or what it is for a whole to be

[1] *Ἔχει δ' ὁμοίως ἥ τε τῶν ὀστῶν καὶ ἡ τῶν φλεβῶν φύσις. Ἑκατέρα γὰρ αὐτῶν ἀφ' ἑνὸς ἠργμένη συνεχής ἐστι, καὶ οὔτε ὀστοῦν ἐστιν καθ' αὑτὸ οὐδέν, ἀλλ' ἢ μόριον ὡς συνεχοῦς ἢ ἁπτόμενον καὶ προσδεδεμένον, ἵνα χρῆται ἡ φύσις καὶ ὡς ἑνὶ καὶ συνεχεῖ καὶ ὡς δυσὶ καὶ διῃρημένοις πρὸς τὴν κάμψιν.*

one single thing. Notice how the notion of continuity contrasts with the claim that the bones are separate things. The bones are parts of the whole and not separate because they are attached and in contact with the whole. Moreover, the continuity seems to allow for gradation. The bones may be either one and continuous, or they might be seen as two when flexed.

This is only one of many such examples that one could quote. But I think that it becomes clear that a zoologist must be at a loss, if she has not a sufficient grasp of concepts such as continuity or contact. Without these concepts she is in exactly the same position as the scientist who studies the movement of animals, but does not have the concept of place at her disposal. In this sense, an account of body is indispensable for a zoologist. However, in exactly the same sense as the study of place, the study of continuous and extended wholes, of bodies and their surfaces is not part of zoology, but of the conceptual underpinnings of zoology.

3.2. The Connection Between *Physics* III–IV and the Study of Body

3.2.1. *A remark on* Physics *III–IV*

The third and the fourth books of the *Physics* contain five essays, on the topics of change, the infinite, place, void, and time. The discussion of *Physics* III–IV is, in a sense, not part of physical science, but rather is a preliminary investigation necessary for pursuing this science. They are treatments of central notions that are a prerequisite of doing any branch of physical science.[2] This is why commentators have regularly noted that these essays are not part of physical science, but rather part of 'dialectic'[3] or 'philosophy'.[4]

What these commentators mean, I believe, is that Aristotle's treatment of concepts such as motion, time, and place does not constitute a separate branch of physical science. There is no branch of physical science that could properly be called 'theory of motion' and which is on a par with botany or zoology. The importance of the discussion of *Physics* III–IV is therefore not in establishing unknown branches of physical science, but rather in making these concepts available to the working physicist. If this is right, then these essays are more principled discussions of notions that are drawn upon in the particular branches of physical science.

Physical science concerns things that have an internal principle of motion and rest in them.[5] Therefore, it is crucial for the success of any branch of physical science that the phenomenon of motion is well understood. Or in explaining *where* fire is physicists use the concept of place, and in explaining *how long* it takes for an

[2] Of course, not all of these notions are employed in physical science. The point of the discussion of void is arguably to show that this notion is inconsistent and should therefore *not* be used in physical science. But it still can be seen as a prerequisite for doing physical science because it enables the physicist to know why she should not use this notion in the specific branches of physical science.
[3] Hussey 1983, ix. [4] Owen 1986, 242. [5] *Ph.* II.1 192b8–193a2; III.1 200b12.

animal to reach maturity they use the concept of time. If any of these notions were incoherent, these explanation would be empty. Indeed, an explanation of the growth of an animal through an appeal to the void contained in its matter is flawed partly because the notion of void is plagued by difficulties and should not be used in explanations in physical science. Therefore, it is important for the physicist to have an understanding of these notions when she goes about doing science. The need for these conceptual underpinnings, for a philosophical discussion, broadly understood, of the key concepts of physical science, can be compared fruitfully to modern discussions about the nature of space or time:

> Most of the time the reader will not be far out if he takes Aristotle to be doing in these essays much what a modern physicist or philosopher does when discussing certain very general topics, such as the nature of time or space, in general terms. (Hussey 1983, ix)

It is equally clear that these essays do not contain discussions of any old topics, but rather of those that are crucial for physical science. The study of bodies and magnitudes, I claim, should be ranked among these concepts. An account of bodies and related notions is something that the physicist cannot do without.

This, then, is the crucial point of comparison: motion, the infinite, place, and time are part of the conceptual underpinnings of physical science. Any physicist will use them extensively while pursuing her specific field of study. These notions are indispensable for the working physicist. And in the same way should the study of body be seen as indispensable for the physicist and part of the conceptual underpinnings of physical science. This, however, is compatible with the assumption that the theory of body is—just as the study of motion, place, and time—not itself a *branch* of physical science. Zoology or botany are branches of physical sciences. They constitute unified fields of scientific study which have one genus as their subject matter. This need not be true of the theory of body or, for that matter, the theory of motion, place, or time. In the latter cases the discussion is one of principles which provides resources that can then be used in the particular branches of physical sciences. In this sense, the propositions which are set up in Part II should be seen as propositions that can be employed in the specific fields of physical science without themselves constituting a specific branch of physical science.[6]

3.2.2. *Body as part of the conceptual underpinnings of physical science*

Why should one think that the theory of bodies is on a par with the study of place and motion? One reason is that we find an explicit statement to this effect:

> Since the science of nature is concerned with magnitudes and motion and time, and each of these is necessarily either infinite or finite, it is incumbent on the student of nature to discuss the infinite.[7] (*Ph.* III.4 202b30–35)

[6] For the complete list of the propositions as well as the purpose of those propositions see Appendix B.
[7] *Ἐπεὶ δ' ἐστὶν ἡ περὶ φύσεως ἐπιστήμη περὶ μεγέθη καὶ κίνησιν καὶ χρόνον, ὧν ἕκαστον ἀναγκαῖον ἢ ἄπειρον ἢ πεπερασμένον εἶναι, (...) προσῆκον ἂν εἴη τὸν περὶ φύσεως πραγματευόμενον θεωρῆσαι περὶ ἀπείρου.*

In terms of the indispensability of such a treatment for physical science, Aristotle suggests that a discussion of magnitudes is on a par with the discussion of place, time, or motion.[8] Thus, a discussion of body and magnitudes should be connected to and placed alongside the discussion of *Physics* III–IV.

As I said, it is true that as a matter of fact we do not find a self-contained treatment of body and magnitude as we do in the case of motion, the infinite, time, or place which may lead to the assumption that the topic is not important for Aristotle. So let me elaborate a little further on it and provide some more systematic reasons to take the quotation above at face value.

I think the reason why Aristotle discusses the concepts in *Physics* III–IV that he in fact discusses is their intimate connection to motion. For several reasons, the concept of motion is crucial for physical science. As already mentioned, one of the most important reasons is that physical science is concerned with substances that have a nature, that is, an internal principle of motion and rest. And as Aristotle makes clear, motion cannot be properly understood without place, time, and the infinite.[9]

Yet, this does not imply that the study of bodies, or more generally, magnitudes has no significance for the project of the *Physics*.[10]

First, as already noted, all physical substances are bodies. Thus, if there is some serious trouble with the notion of a three-dimensionally extended and continuous object, if say, something akin to Zeno's paradoxes threatens, then the whole project of studying physical substances cannot get off the ground at all. This is indeed a serious threat if we think of later sceptical discussions such as the one by Sextus Empiricus who argued that the very notion of a body is inconsistent.[11]

Second, the concept of bodies and extended magnitudes is intrinsically linked to other concepts in *Physics* III–IV. Thus, there is a connection to the discussion of motion and the other concepts. This is most obvious in Aristotle's discussion of the infinite and his theory of place. In the course of these discussions he makes extensive use of notions that belong to a treatment on magnitudes. The discussion of the infinite relies heavily on the concept of a division of a magnitude, a magnitude that is described as a composite of extension and topological limit. And, as Aristotle emphasizes, his main interest is in the question whether there is an infinite perceptible magnitude, that is, a magnitude belonging to a physical substance, and the

[8] Aristotle refers with the word 'magnitude' to physical quantities, as Hussey 1983, 73 rightly remarks. In general, the study of magnitude, time, and motion is a study of quantities. This is suggested by *Categories* 6 and *Metaphysics* V.13 (I discuss these texts in Chapter 5 and Appendix A). That they must be physical quantities is suggested by the argument. For the magnitudes are, as Aristotle says, either finite or infinite. Therefore, he concludes, the physicist has to consider whether the infinite exists and, if so, in what way. This conclusion evidently only makes sense if one assumes that the physicist studies extended magnitudes.

[9] *Ph.* III.1 200b12–25.

[10] For now I will focus mainly on the indispensability claim. In Section 3.3 I argue that this study is distinctly physical.

[11] S.E. *M* 9.359–440. For a recent discussion of this text, which also deals with its implications for our present topic, see Betegh 2015.

ramifications for physical science which the answer to this questions has.[12] Place, on the other hand, is defined as the inner limit of the container that is in contact with the contained. Again, this requires an understanding of what a limit is, how it connects to the body of which it is the limit, and how limits are individuated. For example, how can the limit of the container be distinguished from the limit of the contained, if the limits coincide? It is obvious that an answer requires a general understanding of the concepts involved and their ontology. The list could be extended, but I think that reflection on these examples already show that central questions of Aristotle's theory of the infinite, place, or time, and motion cannot be answered without a grasp of the mereotopological concepts that are at the heart of this study.[13]

It is clear, then, that the study of place, of the infinite, and related concepts relies on the study of magnitudes. If physical science cannot get started without a discussion of the concepts of place, time, infinity, etc., which, in turn, presuppose the concept of body, it is clear that physical science cannot get started without an analysis of the notion of body. Aristotle, we may say, owes us an account of body and related notions because this account is necessary for an account of the basic and fundamental notions employed in physical science. Thus, it comes as no surprise that commentators have, in discussing place, time, motion, or the infinite, employed concepts and touched upon issues that arise in the study of extended bodies. In this study I aim to provide the systematic underpinnings of their observations.[14]

3.2.3. *Further evidence from* On Generation and Corruption

We can underline the findings of *Physics* III.4 by considering two further passages from *On Generation and Corruption* that support the view that an account of bodies and related notions is indispensable for all branches of physical science.

Action and passion are, without any doubt, key concepts for doing physical science. However, the notions of action and passion presuppose an account of contact:

But if we must investigate action and passion and mixture, we must also investigate contact. For action and passion (in the proper sense of the terms) can only occur between things which

[12] Cf. *Ph.* III.5 204b1–4.

[13] It may be instructive to note that *On Location* by Ben Morison includes a discussion of these concepts (cf. Morison 2002, 139–46) and in their review Casetta and Varzi focus almost exclusively on these mereotopological questions (cf. Casetta and Varzi 2005).

[14] This is also the reason why I wish to connect the theory of body and magnitude to *Physics* III–IV, rather than *De Caelo*. I do not wish to dispute that the *De Caelo* is an important work for the study of bodies and especially *De Caelo* I.1 will be discussed extensively. But, as said above, what I mean by 'theory of body' is not the study of the four elements which Aristotle calls 'primary bodies'. A theory of body or, more generally, extended magnitudes is a theory of what belongs to physical objects insofar as they are bodies, i.e. three-dimensional extended entities. Accordingly, a theory of bodies includes, e.g. a theory of what it is to be a limit or deals with the question as to whether all bodies are divisible into atoms or not. These questions are not part of a theory of the elements in a more restricted sense.

can be in contact with one another; nor can things enter into a mixture at all unless they have come into a certain kind of contact.[15] (*GC* I.6 322b21–5)[16]

Since contact is a relation between extended magnitudes, an account of bodies and extended magnitudes and related mereotopological notions is indispensable for the project of physical science.

Moreover, the physical scientist employs in her study the notions of generation, qualitative change, or growth and decay. Surely, almost no branch of physical science can be done without these notions. Again, the explication of these notions crucially depends on one's view of the structure of bodies and magnitudes:

The starting-point, in dealing with all these difficulties, is this: 'Do things come-to-be and alter and grow, and undergo the changes contrary to these, because the primary things are indivisible magnitudes? Or is no magnitude indivisible?' For this makes the greatest difference. And again, if there are indivisible magnitudes, are these bodies, as Democritus and Leucippus maintain? Or are they surfaces, as is asserted in the Timaeus?[17] (*GC* I.2 315b24–30)

If all physical magnitudes are ultimately composed of indivisible atoms, the account of generation and destruction will differ considerably from an account that works on the assumption that bodies are infinitely divisible. Both accounts, in turn, will differ from a third account that assumes that all physical bodies are ultimately decomposable into surfaces. The way in which Aristotle contextualizes the argument in *GC* I.2 makes clear that the importance of this question does not derive from mathematical considerations (though it is, of course, important in mathematics, too). Rather, the underlying topology of bodies, whether they are infinitely divisible into bodies or rather ultimately decompose into atoms or surfaces, earns its importance through the implication it has for a proper account of generation and corruption. It is part of the conceptual underpinnings of physical science to have a proper account of bodies and magnitudes. If we do not, we risk ending up with a fundamentally flawed account of central notions of physical science. Or, as Aristotle says in one of his few ironic remarks:

Admit, for instance, the existence of a minimum magnitude, and you will find that the minimum which you have introduced causes the greatest truths of mathematics to totter.[18]
(*Cael.* I.5 271b9–11)

[15] *᾿Αλλὰ μὴν εἰ περὶ τοῦ ποιεῖν καὶ πάσχειν καὶ περὶ μίξεως θεωρητέον, ἀνάγκη καὶ περὶ ἁφῆς· οὔτε γὰρ ποιεῖν ταῦτα καὶ πάσχειν δύναται κυρίως ἃ μὴ οἷόν τε ἅψασθαι ἀλλήλων, οὔτε μὴ ἁψάμενά πως ἐνδέχεται μιχθῆναι πρῶτον.*

[16] Cf. *Ph.* VII.2 which contains an elaborate argument that all motion requires contact between mover and moved.

[17] *᾿Αρχὴ δὲ τούτων πάντων, πότερον οὕτω γίνεται καὶ ἀλλοιοῦται καὶ αὐξάνεται τὰ ὄοντα καὶ τἀναντία τούτοις πάσχει, τῶν πρώτων ὑπαρχόντων μεγεθῶν ἀδιαιρέτων, ἢ οὐθέν ἐστι μέγεθος ἀδιαίρετον· διαφέρει γὰρ τοῦτο πλεῖστον. Καὶ πάλιν εἰ μεγέθη, πότερον, ὡς Δημόκριτος καὶ Λεύκιππος, σώματα ταῦτ' ἐστίν, ἢ ὥσπερ ἐν τῷ Τιμαίῳ ἐπίπεδα.*

[18] *Οἷον εἴ τις ἐλάχιστον εἶναί τι φαίη μέγεθος· οὗτος γὰρ τοὐλάχιστον εἰσαγαγὼν τὰ μέγιστ' ἂν κινήσειε τῶν μαθηματικῶν.*

Notice that the importance of a correct account of the underlying topology of bodies and magnitudes for physical science does not necessarily mean that it is the task of the physicist to work out the proof that bodies are infinitely divisible. In fact, one way to take the quotation is that the proof that bodies are infinitely divisible is probably worked out by the mathematician.[19] I am inclined to read the passage differently. It merely says that a mistaken physical account may contradict established mathematical theorems, which, of course, it shouldn't. Be that as it may, here I wish to emphasize two points: first, the physicist is concerned with the question of infinite divisibility; and second, although the importance of a study of magnitudes for the conceptual underpinnings of physical science does not entail that in this study physical consideration *alone* play a role, a study of body has a distinctly physical character. This is the topic of Section 3.3.[20]

3.3. A Physical Study of Body

Having placed the study of body within the realm of physical science, one may wonder whether it should not better be seen as a part of mathematics. After all, Aristotle himself suggests that mathematics is the study of lines, surfaces, and bodies.[21] It is true that the study of magnitudes is in this respect unlike the study of place, time, or motion. The latter seem obviously to be concepts only dealt with by the physicist.[22] The study of magnitudes, however, cannot be so firmly placed in the context of a single science. Aristotle, I believe, is aware of this. The correct answer to the question, 'who it is that studies magnitudes, the mathematician or the physicist', is 'both'. As I will show in Chapter 4, Aristotle explains in *Physics* II.2 how this is possible. Both the mathematician and the physicist study magnitudes, but not *qua* the same. The physicist studies magnitudes insofar as they belong to physical substances. In this sense, she studies magnitudes *qua* physical. The mathematician, on the other hand, studies them not insofar as they are physical. She studies magnitudes as if they were separate. However, since sciences are individuated by their manner of investigation and their respective focus, physical science and mathematical science are distinct

[19] Cf. also *Cael.* III.1 299a1–11 which contains a reference to *Physics* VI.1 and says that the refutation was part of the analysis of motion. It is unclear whether *Physics* VI.1 should be seen as a mathematical argument. I think that it should for reasons that will be fully explained only later (part of the reason is that the account of contact differs from the canonical statement in *Physics* V.3). See Section 6.4.5.

[20] In Chapter 4, I will argue that in the account of magnitudes the physicist may draw on mathematics too. But, of course, it remains true that it is first and foremost the task of the physicist to provide a theory of the ontology of bodies.

[21] *Metaph.* XIII.3.

[22] I intentionally use the word 'seem' here because it is not clear whether this is true. The astronomer, for example, deals with moving things. In the science of astronomy, it seems, motion-predicates occur. It is, however, not clear whether astronomy should be placed under mathematics or physics. Simplicius in his commentary on *Physics* II.2 argues that astronomy is part of mathematics. For a discussion see Mueller 2006.

sciences. Though concerned with the same underlying reality, the way in which the mathematician and the physicist study magnitudes explains why they are engaged in two distinct sciences.

To be sure, the physicist may use theorems established in mathematics. The mathematician might show that no line is composed of points, and the physicist could draw on this result and conclude that there are no bodies made up of indivisible atoms.[23] It is, however, crucial to see that this does not imply that the study of magnitudes really is part of mathematics or that the part of physics dealing with body and other magnitudes is really a branch of mathematics. It is a mistake to assume that Aristotle's arguments are mainly mathematical and simply carried over to the realm of physics. The focus on this paradigm has, in my mind, misled commentators. They jump from the fact that Aristotle employs mathematical arguments to the conclusion 'that [it] is fair to say that Aristotle's conception of continuous quantity or magnitude is a geometrical conception' (White 2009, 266). I disagree with this conclusion, if it means that Aristotle's arguments should be considered as mathematical by his own standards. Some of them might be, but surely not all. It is plausible, I believe, to assume that in giving an account of bodies the physicist can draw on several sources and forms of argument. Some of them are mathematical, but most are distinctively physical arguments.

Therefore, I think it is important to outline here a distinct form of argument that the study of magnitudes involves over and above mathematical arguments. This form of argument specifically asks what belongs to body insofar as it is the body of a physical substance.[24] It is distinctly physical because it considers the mereotopological properties of body *qua physical*.

[23] This, I believe, is the lesson to be learned from *Cael.* III.1 and *GC* I.2. See also Section 4.2.2.

[24] One might distinguish a further form of argument that might be called 'topic-neutral' or 'logical'. This form of argument relies on a quite general conception of bodies and magnitudes. As such, the conception is neither part of mathematics nor of physics, but rather of dialectic. In this way it connects to my earlier remarks about the study of magnitudes belonging to the conceptual underpinnings of physics. In calling this form of argument 'logical' I want to capture Aristotle's expression 'λογικῶς'. We already encountered this expression in Aristotle's discussion of the infinite in *Physics* III.4–8. Aristotle announces that his question is whether there is an infinite body among physical substances. He then argues that if all bodies are bounded by a surface, it is impossible that there is an infinite body (*Ph.* III.5 204b4–7). Aristotle calls this a 'logical' argument. What type of argument is that? One might assume that it means it is a 'merely verbal' argument. Interpreted thus, the expression might have a pejorative connotation. The argument is not really an argument, but rather some form of linguistic trick. I think that this interpretation should be resisted, and I agree with Burnyeat 2001, ch. 5 that it means more than 'verbal'. Aristotle does not want to say that the *use* of our words suggests that there cannot be an infinite body. This seems to me to impose an anachronistic reading of Aristotle. I believe that Aristotle intends this as a proper argument. If the argument is successful, it follows that there cannot be an infinite body. The argument is logical because it is topic-neutral. It is not part of any specific science, nor does it hold only of some bodies, be they physical or mathematical. This is indeed suggested by our passage. For Aristotle explicitly says that the argument implies that there can be neither a sensible nor an intelligible body which is infinite. A similar point could be made about the discussion of body in *Cael.* I.1 (which seems to be the position of Falcon 2005, 38).

In this sense it might be connected to the proposal by Owen 1986 that 'λογικῶς' is connected to conceptual analysis. A similar position is taken by Frede and Patzig 1988, who hold that logical is used by Aristotle in order to characterize investigations as linguistic, conceptual, as opposed to scientific. Nevertheless, Aristotle does not in any way dispute the importance of such formal analyses. One must only be aware that they need to be supplemented with contentful considerations. The last point is important. A logical argument is an argument. For the meaning of 'logical' see also Smith 1997b, 90–2.

3.3.1. Why the study of bodies belongs to physical science

There are two arguments why the study of body that Aristotle carries out is physical most of the time rather than mathematical in nature. First, although parts of physical science can draw on results in mathematics, this does not make the enterprise mathematical. To return to the example of *GC* I.2: It is the task of the physicist to show that no physical substance can be composed of atoms or be divided into non-magnitudes. This is a theorem about physical substances insofar as they are physical bodies. The mathematician is not concerned with this theorem. There is no theorem in mathematics stating that physical bodies cannot be composed of atoms or be divided into non-magnitudes. There might be a theorem in mathematics stating that no line can be composed of points. In the course of her argument the physicist can draw on this mathematical theorem, but it is clear that her argument is in the context of physical science. This, however, is not a shortcoming on either side. The physicist should not prove the mathematical theorem, nor should the mathematician apply her theorems to physical reality.

Second and more important, the study of physical substances *qua* physical bodies contains many results that could not possibly be mathematical results. The reason is its intimate connection to ontological considerations about bodies and magnitudes and their relation to the physical substance to which they belong. The physicist studies bodies and magnitudes as bodies and magnitudes *of* physical substances.[25] The mathematician, on the other hand, treats quantities as if they were separate. As I will argue later, the mathematician studies a body only insofar as it is a three-dimensionally extended quantity. She does not take into account that there is no such body ontologically separate from physical substances. But she is not concerned with the underlying ontology of these items. In this the study of the physicist differs. The physicist takes into account that it is the body of a physical substance.

To illustrate the point consider, for example, the definition of boundary and figure that we find in Euclid:

13. A boundary is that which is the extremity of something.
14. A figure is that which is contained by some boundary or boundaries.[26]

(Euc. I. Def. 13 and 14)[27]

Equipped with these definitions the mathematician knows what a boundary is and what a figure is. On the basis of these definitions she can then define circles, rectilinear figures, etc. But it would be awkward to ask the mathematician how we should conceive of the ontological relation between the boundary and that which it bounds. The mathematician can do her study quite independent of the question whether the lines which are the boundaries of a rectangle are ontologically dependent on the body.

[25] This is, I believe, also a lesson to be learned from *Physics* II.2. See Section 4.1.

[26] *ιγʹ. Ὅρος ἐστίν, ὅ τινός ἐστι πέρας. ιδʹ. Σχῆμά ἐστι τὸ ὑπό τινος ἤ τινων ὅρων περιεχόμενον.*

[27] All translations of Euclid by Fitzpatrick 2007.

Those questions, I suggest, do not arise *within* mathematics. Rather, they arise outside mathematics as part of the foundation of mathematics or a general ontological study of magnitudes. But in proving theorems the mathematician does not need to consider such questions.

The physicist, on the other hand, must consider these questions or at least be aware of them. Ontological considerations, like the question whether boundaries depend on their host, must be taken into account by the physicist. The reason is, as I said, that the physicist studies lines, surfaces, or bodies *as belonging to* a physical substance. Questions about ontological priority of bodies over surfaces naturally occur in a discussion of physical substances and their attributes. Physical substances are three-dimensionally extended and are bounded by surfaces. The surfaces, Aristotle argues, are ontologically dependent on the physical substance they bound.[28] Thus, if the physicist studies bodies and magnitudes insofar as they are the bodies and magnitudes of a physical substance, the concept of ontological dependence plays a role. None of these insights could be part of mathematics, simply because in mathematics the concept of physical substance is not used at all.[29]

There may also be structural differences.[30] Consider the division of a line. Let us suppose that the line Z is divided at point X into the discrete halves A and B. Does X belong to A or B after the division? Moreover, if X belongs to A, does B have two endpoints? Mathematically, the first question is answered by stipulation. There is no fact of the matter whether X belongs to A or B. Moreover, if X belongs to A, then B does not have two endpoints, but it corresponds to a half-open interval. However, it is doubtful whether these answers make sense in the case of *physical* lines. If Z is a physical line (assuming that there are one-dimensional lines in the physical world), can it really be a matter of stipulation whether X belongs to A or B? Or does it make sense to say that there is a line without an endpoint? Typically, we assume that a physical line has two endpoints. Thus, there is a real question whether mathematical structures fit the physical structures. Aristotle, as I will argue, distinguishes between the mathematical and the physical case and answers these with respect to *physical* lines in this way: in cases of division there is no stipulation, but rather *two* new endpoints come into existence. Also, there are no open or half-open intervals in the physical world, but every line has two endpoints that belong to the line.

Commenting on a similar example in a discussion of Zeno's paradoxes Mark Sainsbury writes:[31]

[28] I will discuss these questions in Section 6.3.4.

[29] This is not to say that the boundaries were always sharply demarcated in practice. In many discussions of Platonic or sceptical provenience ontological and mathematical considerations are not duly separated. See, e.g. Sextus Empiricus *M*3 and *M*9. My point is that the boundaries are sharply demarcated in Aristotle's theoretical conception of the sciences.

[30] I discuss this case in Section 6.3.

[31] I want to thank Joseph Bjelde for bringing this book to my attention. Sainsbury 2009, 15–18 makes exactly the same distinction between a mathematical and a physical approach.

> In these remarks, I have assumed that we have coherent spatial notions, for example, that of (two-ended) length, and that if some mathematical structure does not fit with these notions, then so much the worse for the view that the structure gives a correct account of our spatial notions. (Sainsbury 2009, 18)

I think that this comment is very apt and that the failure to see the potential differences between mathematical and physical structures has been part of the reason why the study of magnitudes as a prerequisite for doing physical science has been unduly neglected. In the case of infinite divisibility, a—by modern standards—mathematical approach might lead to impressive results. But for many of the other concepts involved in the study of magnitudes, concepts such as continuity, boundary, or position, an overly mathematical approach might be detrimental. Surely, this does not yet show that we have, in fact, coherent spatial notions which are applicable to the physical world. But it shows that we cannot undertake this task by relying on mathematical considerations alone. And Aristotle, as I argue, is aware of this.

3.3.2. More exemplary cases

Considering two further examples will, I hope, help to illustrate the distinctive nature of the study of body and magnitude and why it is important for physical science. Both examples will be discussed in detail in Part II, Section 6.4 and Chapter 7.

First, any particular physical substance has a certain extension, a *diastema* as Aristotle calls it. The extension can, roughly speaking, be conceived as that which lies within the limits of the magnitude or body of the physical substance. In virtue of having a certain extension the physical substance has a certain length, breadth, and depth. In short, it can be measured. But what is this extension, ontologically speaking? Does the extension belong to the body and the physical substance, or is it independent of it? This question is not irrelevant for the physicist. For, if the extension is independent, this affects the view one might have about void. A void can be conceived of as an independent extension that is unoccupied. If extension is independent of physical substances, it should be possible that there is a void. Moreover, a place according to this conception could be seen as occupied extension. Thus, when the physicist says that a substance has a certain extension, this is explained by the extension that the body of the substance occupies, but the extension itself is separate from the body. However, things radically change if there is no independent extension. In an explanation of the extension of a physical substance one would rely on features of that substance. Extension is not something over and above objects. Rather, it is, as I will argue in detail, an abstract way of considering the (ordinary) matter of an object.

Second, the notions of contact and continuity are arguably at the heart of thinking about extended objects. I have already quoted passages from the *Parts of Animals* and *On Generation and Corruption* where these notions play a central role.[32] As will

[32] See Sections 3.1 and 3.2.3.

emerge later, the relation of contact holds between two independent objects, whereas the relation of continuity holds between the parts of a single object. The notion of continuity is intimately tied to the idea of a causal factor that unites the parts into a single object. But the idea of a causal connection between the parts of a single object is clearly an idea that has its place in physical considerations. If the physicist employs the notion of continuity to explain the topological integrity or connectedness of an object, she does so by invoking *inter alia* a certain causal factor that explains this form of unity.[33] Thus, the concept of continuity as it is used in the study of bodies and magnitudes that I envisage is metaphysically loaded. It is essentially tied to the concepts of unity and causality. Consequently, in the explanation of the continuity of the body of an animal or of the parts of the body of an animal with each other, the physicist will necessarily rely on considerations about the unity, nature, and form of that animal. Her explanation will without doubt fail if she is not aware of the ontological implications the concept of continuity has.

As I said, the details of my interpretation will be made much more precise in Part II of this work. Let me here instead draw attention to a general and important trend in these arguments: all of these arguments are arguments about magnitudes insofar as they are the lines, surfaces, or bodies *of* physical substances. To provide the conceptual underpinnings of physical science the physicist must include ontological considerations in her account of body. These ontological consideration are not idle speculations that the physicist might dispose of. Rather, they are central to an adequate account of body and magnitudes.

This then is the context in which the results of the second part of my work should be seen. In her study the physicist does more than consider 'the shape of the moon or the sun' (*Ph.* II.2 193b29). Of course, this is the task of the physicist, too, but by no means does it fully describe the method in which the physicist studies magnitudes *qua* physical.

3.3.3. The limits of the study of physical bodies

In the previous section I have argued that there is a distinctly physical study of body insofar as it is the body of a physical substance. How should one delineate such a study? How to distinguish between what belongs to physical substances *qua* physical bodies and what belongs to physical substances *as such*? Implicitly, I will delineate the content of the study in Part II.[34] Nevertheless, let me outline a more explicit answer here. In its most general form, the study of bodies takes into account only features of physical substances which they have in virtue of being bodies. This, of course, does not enable us to delineate the study because it only invites the further question 'How do we single

[33] Aristotle illustrates this idea with the example of a shoe. *Metaph.* V.6 1016b11–16.

[34] There is one topic which I will not address in this work, although it may justifiably be thought to be part of a theory of bodies. This the question why two bodies cannot be in the same place at the same time. I have discussed this question in detail in Pfeiffer 2016.

out these features?' But it provides a heuristic in the sense that each candidate-feature should be tested whether physical substances have it in virtue of being bodies. Since the task of checking each candidate-feature cannot be accomplished here, let me apply this general idea to the particular case of causal efficacy, which, I think, is a prime example.[35]

Here is how one might reason: action and passion require contact,[36] hence, being causally efficacious is something that belongs to physical substances insofar as they are physical bodies. This impression may be reinforced if one thinks, for example, of the Epicurean definition of body as that which is three-dimensionally extended and has *antitupia*.[37] *Antitupia* distinguishes bodies from void and explains why bodies do not move through each other, but collide when they meet. Thus, according to an Epicurean theory, the causal efficacy of physical substances seems, at least partly, to be a direct consequence of their being bodies.

For Aristotle, I suggest, this is not the case. The causal efficacy of physical substances is not due to their being bodies. It does not derive from their being extended, continuous, or bounded that physical substances act upon each other. This is a necessary, but not sufficient condition. Causal efficacy derives from having contrary properties in the same genus.[38] At the basic level, these are the pairs of elemental qualities hot–cold and wet–dry. The elemental qualities belong to things insofar as they are physical *substances*, not insofar as they are bodies. Although all physical substances are, in fact, bodies and thus it is true that only bodies possess the elemental qualities, the elemental qualities are not themselves bodily properties that are comparable to features such as, for example, continuity.

Moreover, the comparison with the Epicurus's view of bodies and their causal efficacy suggests a general observation: whether causal efficacy is part of the conception of physical substances *qua* bodies or not depends on the broader commitments and specifics of the theory in question. There is no single answer whether a theory of body should include such a discussion. For Epicurus it does, for Aristotle it does not. This divide, I suggest, runs through the history of philosophy. Causal efficacy belongs to the conception of body *as such* for Locke and Leibniz, for example, but not for Descartes who is closer to Aristotle in that respect.

[35] I want to thank Gábor Betegh for pressing me on this point.

[36] Cf. *GC* I.6 322b21–5, quoted in Section 3.2.3.

[37] Cf. S.E. M 1.21.

[38] *GC* I.7 323b29–24a11.

I will discuss Aristotle's conception of the extension or matter of bodies (Section 6.4). In this sense, Chapters 5 and 6 together will yield a systematic and detailed theory of bodies.

Chapter 7. In this chapter I introduce the important distinction between continuity and contact. This is an addition to the previous chapters because it enables us to make a topological distinction between the way in which several objects are connected and the way in which the parts of a single object are connected. In this sense, I shift from a description of the topological features of a single body to an analysis of the interrelations between bodies (Section 7.2).

Moreover, I will argue that the topological difference between continuity and contact—the number of boundaries involved—is grounded in the ontological difference between ontologically independent objects and parts of a single object that are not ontologically separate. Continuity is thus connected to the unity of an object. The spatial wholeness and continuity of a body is explained through considerations about the metaphysical status of the body (Section 7.3).

Finally, in Section 7.4, I turn to a more detailed examination of the connection between continuity and unity. The continuity of an object is, I will argue, explained by its form. In this sense, the continuity of an object is grounded in a causal account of its unity. Although no complete account of the ontology of physical substances and their forms will be offered, the section will provide an account of how the study of bodies relates to an overarching analysis of physical substances.

5

Body in *Categories* 6

5.1. Introduction and Framework

Aristotle begins his discussion of the category of quantity in *Categories* 6 by introducing the following classificatory scheme.

> Of quantities [1] some are discrete, others continuous; and [2] some are composed of parts which have position in relation to one another, others are not composed of parts which have position.[1] (*Cat.*6 4b20–23)

Aristotle presents two ways of classifying quantities. On the one hand [1] quantities divide into the continuous and the discrete and on the other [2] into those whose parts have a position and those that do not. Aristotle does not tell us how the two classifications are connected. Rather he discusses both classifications one after the other without explicitly connecting the two.[2] But since Aristotle later claims that 'only these we have mentioned are called quantities strictly, all the others derivatively'[3] (*Cat.*6 5a38–39), we might at least be confident that Aristotle believes that his discussion of quantities is complete and that he has mentioned all of them. That is to say, the discussion which begins at 4b20 and ends at 5a37 contains an exhaustive list of all quantities, and each quantity can be brought under one of the classificatory schemes. The main task in the next sections is to understand the classificatory schemes and to place body within them.

Aristotle proceeds as follows: *given* a list of quantities, some of these are continuous and others discrete, some are composed of parts with position, and others of parts without position. Of course, this leads to the question why Aristotle seems confident that he has in fact covered all quantities. Aristotle has not given sufficient grounds for believing this.[4] However, since for our purposes the classificatory schemes and the classification of body and the basic magnitudes are relevant, the question whether the

[1] *Τοῦ δὲ ποσοῦ τὸ μέν ἐστι διωρισμένον, τὸ δὲ συνεχές· καὶ τὸ μὲν ἐκ θέσιν ἐχόντων πρὸς ἄλληλα τῶν ἐν αὐτοῖς μορίων συνέστηκε, τὸ δὲ οὐκ ἐξ ἐχόντων θέσιν.*

[2] [1] is discussed in *Cat.*6 4b24–5a15 and [2] in *Cat.*6 5a15–37.

[3] *Κυρίως δὲ ποσὰ ταῦτα μόνα λέγεται τὰ εἰρημένα, τὰ δὲ ἄλλα πάντα κατὰ συμβεβηκός.*

[4] An example of a quantity that might be missing is motion. In *Metaph.* V.13 motion appears together with time and place as a derivative quantity. Since time and place are listed as proper quantities in *Categories* 6 one could reasonably argue that in the *Categories* motion should be treated on a par with time and place and hence as a per se quantity. For an interpretation of *Metaph.* V.13 see Appendix A.