6

A Topological Conception of Bodies

6.1. Introduction

Categories 6 yields a systematic theory and classification of body.[1] Aristotle introduces fundamental properties such as continuity, extension, position. These concepts are the building blocks for a theory of extended objects. Thus, I believe that a study of this text is fundamental for any study of quantities and extended objects. But the theory presented there is, as I said, not complete. It provides important background for understanding Aristotle's theory of extended objects, but it does not answer all of the questions. It is never the case, of course, that all of the questions are answered. So let me be more specific on the argument of this chapter: first, Aristotle believes that three-dimensional objects are complete and perfect in virtue of having three dimensions. The *Categories* allow for a classification of continuous quantities and body, but there is no indication that bodies are privileged. Aristotle claims, however, that they are. I will discuss Aristotle's claim that bodies are complete in Section 6.2. Second, Aristotle suggests in the *Categories* that lower-dimensional entities are boundaries of higher-dimensional objects. The boundaries at which the parts of a body are connected are planes. But what are boundaries? How should we conceptualize them? Is there a distinction between the outer limits and the internal limits of an entity? I will address the topic of limits in Section 6.3. Third and closely connected to the previous point, a body can be analysed into its limit and its extension, but what is the latter? What is the ontological status of extension? This is the task of Section 6.4.

6.2. Bodies are Complete

In Part I of this study I claimed that the study of bodies is a part of the conceptual underpinnings of physical science. It investigates what belongs to them insofar as they are bodies *of* physical substances. In this section we will consider a concrete example. Aristotle argues that bodies are complete and perfect in virtue of being

[1] The same is true of *Metaphysics* V.13, which is another text that is rarely studied. I discuss this text in Appendix A.

three-dimensional. If an item is three-dimensional, it is implied that it is prior to an item that is two-dimensional. Additionally, Aristotle claims that because bodies are complete, there cannot be a four-dimensional magnitude. How can this be explained? I think that the best explanation is that certain topological properties are linked to and determined by the nature of the object in question. Even though the content of this claim is applicable to any three-dimensionally extended object, the justification of the claim is based on considerations regarding the nature of bodies. The claim about topological and quantitative features is based on the ontology and nature of physical bodies. This, I propose, is argued for by Aristotle in *De Caelo* I.1.[2]

6.2.1. De Caelo *on the completeness of bodies*

Bodies are exceptional among extended objects. Bodies are complete and perfect (*τέλειον*)[3] because they have all possible dimensions. Accordingly, lines and surfaces are incomplete. They do not have all possible dimensions and are posterior to bodies. Aristotle defends this claim in the first chapter of *De Caelo*. He begins the chapter with a specification of the proper subject matter of physical science. Aristotle claims that magnitudes and especially bodies are the primary objects of physical science.[4] They are the primary objects because these bodies and magnitudes are constituted by nature.[5] This preamble, I suggest, puts the following discussion in the context of physical science.

Aristotle then goes on to say:

> Of magnitude, that which is extended in one dimension is a line, that which is extended in two is a surface and that which is extended in three dimensions is a body. There is no other magnitude beyond these, since the three (dimensions) are all and the thrice is in every way.[6]
>
> (*Cael.* I.1 268a7–10)

In the first sentence Aristotle mentions the previously discussed definition of magnitudes according to their dimensions. But in the next sentence, Aristotle makes a stronger claim. He claims that there cannot be other magnitudes besides these. There can be no other magnitude because body, by being extended in three dimensions, is extended in *all* dimensions. Aristotle grounds his claim on the shaky evidence that being three implies being an 'all'.[7] As it stands, the argument seems deeply flawed

[2] In my discussion I rely on Betegh et al. 2013. The paper presents a full analysis of the difficult and puzzling first chapter of *De Caelo*. Since I cannot do justice here to the complexities of the chapter, especially the striking allusions to the Pythagoreans, I refer the interested reader to this paper.

[3] 'Complete' or 'perfect' are my translations of *τέλειον*. Cf. Section 6.2.1.2 in this chapter for a discussion of the precise meaning of the term.

[4] *Cael.* I.1 268a1–4.

[5] *Cael.* I.1 268a4–6.

[6] *Μεγέθους δὲ τὸ μὲν ἐφ' ἓν γραμμή, τὸ δ' ἐπὶ δύο ἐπίπεδον, τὸ δ' ἐπὶ τρία σῶμα· καὶ παρὰ ταῦτα οὐκ ἔστιν ἄλλο μέγεθος διὰ τὸ τὰ τρία πάντα εἶναι καὶ τὸ τρὶς πάντῃ.*

[7] 'For, as the Pythagoreans say, the world and all that is in it is determined by the number three, since beginning and middle and end give the number of an "all", and the number they give is the triad. And so, having taken these three from nature as (so to speak) laws of it, we make further use of the number three

because, if we grant that being three implies being an all and being complete, it is hard to explain why there are more than three people in the world. I do not see how Aristotle's argument could be saved from these objections. But perhaps it can be made more understandable by noting that his argument stands in a dialectical context and he seems to rely on premisses that might be shared by him and his opponents.

Be that as it may, Aristotle's discussion of the relation between the nature of physical substances and three-dimensionality is fine-grained and well worth studying, even if the connection between being three and being all proves to be unsatisfactory. It is worth studying because Aristotle employs a model of how three-dimensionality, though not being part of the essence of physical substances, is nonetheless caused by and due to the essence of physical substances.

Before entering the discussion I must emphasize that it is crucial to bear in mind that the whole discussion is set in the framework of physical science. Aristotle even calls the bodies he discusses 'substances' (*Cael.* I.1 268a3). Thus it should be clear that Aristotle's arguments are not independent of physical considerations. The argument is underpinned by considerations about the nature of the bodies in question. It is not a mathematical argument. It is not an argument about three-dimensionally extended objects *in general*. Rather it is an argument about physical bodies. That is to say, it is an argument that concerns the question why physical substances are necessarily three-dimensional. Part of the answer is that by being three-dimensionally extended they have a body that is complete. But the completeness of body in the sense of a three-dimensionally extended quantity can only be understood in the context of that body being the body *of* a physical substance.

Having said that, let us turn to Aristotle's two major claims in the chapter: first, body is exceptional among the existing magnitudes. Second, there can be no other continuous magnitude besides lines, surfaces, and bodies.

6.2.1.1. ARISTOTLE'S ARGUMENTS FOR THE PRIORITY OF BODIES

The first claim, the priority of bodies over lower-dimensional magnitudes, is vindicated thus:

> Therefore, since 'every' and 'all' and 'complete' do not differ from one another in respect of form, but only, if at all, in their matter and in that of which they are said, body alone among magnitudes can be complete. For it alone is determined by the three (dimensions), that is, is an 'all'.[8] (*Cael.* I.1 268a20–24)

in the worship of the Gods. Further, we use the terms in practice in this way. Of two things, or men, we say "both", but not "all": three is the first number to which the term "all" has been appropriated. And in this, as we have said, we do but follow the lead which nature gives' (*Cael.* I.1 268a10–20). For an interpretation see again Betegh et al. (2013).

[8] *Ὥστ' ἐπεὶ τὰ πάντα καὶ τὸ πᾶν καὶ τὸ τέλειον οὐ κατὰ τὴν ἰδέαν διαφέρουσιν ἀλλήλων, ἀλλ' εἴπερ, ἐν τῇ ὕλῃ καὶ ἐφ' ὧν λέγονται, τὸ σῶμα μόνον ἂν εἴη τῶν μεγεθῶν τέλειον· μόνον γὰρ ὥρισται τοῖς τρισίν, τοῦτο δ' ἐστὶ πᾶν.*

Aristotle's thought is hard to grasp, but I suggest taking the passage in the following way:[9] Aristotle takes himself to have shown that being extended in three dimensions is being extended in *all* dimensions because the number three implies being an all. Now he introduces the further thought that being an 'all' also implies a certain kind of completeness and perfection. For, Aristotle argues, the terms 'every', 'all', and 'complete/perfect' do not differ in form. Thus, since body is an all, it must be complete and perfect, too. To justify this inference it is, I think, plausible to assume that 'having the same form' is here used as an equivalent of 'synonymous'.[10] All these terms have the same meaning, and therefore it is permitted to predicate 'completeness' of an item, if it is permitted to predicate 'all' of the same item.[11] Therefore, we can conclude that body is complete, whereas lines and surfaces are incomplete. The latter cannot be complete because they are not extended in *all* dimensions, but only in *some* dimensions.

Having shown that body is unique among the existing magnitudes, Aristotle goes on to argue that that the transition to another genus of magnitude (in this case the fourth dimension) is impossible. This can be seen as the flip side of the coin in Aristotle's argumentation. The completeness and perfection of bodies not only accounts for the priority of bodies over lines and surfaces, but also for the fact that bodies are posterior to nothing.

> One thing, however, is clear. There is no transition to another kind of magnitude, as we passed from length to surface, and from surface to body. For if we could, it would cease to be true that body is complete magnitude.[12] We could pass beyond it only in virtue of a defect in it; and that which is complete cannot be defective, since it is in all ways.[13] (*Cael.* I.1 268a30–b5)

[9] See Betegh et al. for the details.

[10] This is also suggested by the following two parallel passages. The first is *Cael.* I.8 276a32–b4: 'Moreover each of the bodies, fire, I mean, and earth and their intermediates, must have the same power as in our world. For if those elements are named homonymously and not in virtue of having the same form (*μὴ κατὰ τὴν αὐτὴν ἰδέαν*) as ours, then the whole to which they belong can only be called a world homonymously.' The second is *EN* V.1 1129a27–b1: 'Now "justice" and "injustice" seem to be ambiguous, but because the homonymy is close, it escapes notice and is not obvious as it is, comparatively, when the meanings are far apart. For here the difference in form is great (*ἡ γὰρ διαφορὰ πολλὴ ἡ κατὰ τὴν ἰδέαν*). E.g. as the homonymy in the use of *kleis* for the collar-bone of an animal and for that with which we lock a door.' See also Wildberg 1988, 22, who quotes *Cael.*

[11] What does difference in matter amount to? I suggest assuming that for two terms to differ in their matter is for those terms to be predicated of different items. Alexander (according to Simplicius, in *Cael.* 9.5–8) had a similar interpretation. He maintains that 'every', 'all', and 'complete' are the same in form but not with reference to their objects (*ὑποκείμενα*) because 'every' is predicated of a determinate quality, 'all' of continuity, and both of the 'complete'. Thus, 'all' is predicated of masses, like water, whereas 'every' of countable items like horses. In this sense, the meaning of 'all' in the sentence 'She poured all the water out' and the meaning of 'every' in the sentence 'Every person in the room drank a martini' is the same.

[12] This is not the only possible reading of the sentence. Leggatt translates: 'For magnitude of such a kind would no longer be complete' (Leggatt 1995, 49). Thus, he takes *τοιοῦτον* to refer to the hypothetical four-dimensional entity. But *τοιοῦτον* has the same referent as *τέλειον* in line b9 and the latter refers back to 'body'. The sentence does not just mean that something complete cannot be deficient, but that *body* being complete cannot be deficient. Aristotle's main objective is to argue that bodies are complete, and not that a four-dimensional object would not be complete. If we follow Leggatt's reading, this essential connection is lost, because if there were a fourth dimension, a four-dimensional object would be complete (assuming that filling every possible dimension is a mark of completeness).

[13] *Ἀλλ' ἐκεῖνο μὲν δῆλον, ὡς οὐκ ἔστιν εἰς ἄλλο γένος μετάβασις, ὥσπερ ἐκ μήκους εἰς ἐπιφάνειαν, εἰς δὲ σῶμα ἐξ ἐπιφανείας· οὐ γὰρ ἂν ἔτι τὸ τοιοῦτον τέλειον εἴη μέγεθος· ἀνάγκη γὰρ γίγνεσθαι τὴν ἔκβασιν κατὰ τὴν ἔλλειψιν, οὐχ οἷόν τε δὲ τὸ τέλειον ἐλλείπειν· πάντῃ γάρ ἐστιν.*

A transition is possible, if and only if the entity in question is deficient. Bodies, however, are complete and, hence, cannot be deficient. Something complete cannot be surpassed, for the possibility of being surpassed depends on a deficiency of the object in question. Since body is complete, it cannot be surpassed. But if there were another magnitude beyond body, body could be surpassed. Since we have seen that this is not the case, we can conclude that there is no other magnitude.

Aristotle thus holds that bodies are complete, whereas other magnitudes are not. And he believes that because bodies are complete there cannot possibly be another magnitude beyond body. Bodies are unique and singular among magnitudes. They are complete. This, however, raises two questions. First, what precisely does the claim that bodies are complete or perfect (*τέλειον*) mean? Second and more importantly, how does the notion of dimensional completeness relate to the notion of priority in nature?

6.2.1.2. WHAT DOES *TELEION* MEAN?

Aristotle says that bodies are *τέλειον*. I have translated the word as 'complete' or 'perfect'. Aristotle's claim is that bodies, insofar as they are extended in three dimensions, are complete and perfect. The completeness of bodies is due to and grounded in their being three-dimensional. Due to this connection it is natural to assume that being *teleion* means in its first and foremost sense that body is complete because it fills *all* the dimensions that exist. To be *teleion* means being dimensionally complete.

However, I believe that there is more to it. By saying that body is *teleion* Aristotle means more than just stating the dimensional completeness of body. Aristotle is making a normative claim. This normativity is better captured by the word 'perfect'. Body is prior to the other magnitudes. This priority is, I believe, suggested by the semantics of the word '*teleion*'. For Aristotle distinguishes several senses in *Metaph.* V.16: Something is *teleion* if it includes all its parts (that is, is something complete) and if it cannot be surpassed with respect to the excellence proper to its kind (that is, is something perfect).[14] Both senses are relevant in *De Caelo* I.1. Insofar as body is extended in all the dimensions in which a magnitude can be extended, body is a complete magnitude. Insofar as no further magnitude can surpass body, body is the perfect magnitude according to the second meaning of *teleion*.[15] There is also a third sense of *teleion* which is connected to final causation. Insofar as something has reached a (good) end, it is *teleion*.[16]

Wildberg, by contrast, thinks that the attribution of a normative meaning to *teleion* in this context is 'philosophically absurd' because it is 'simply false to say that a body *qua* body is perfect' and 'Aristotle never wanted to claim this'.[17] Wildberg does not specify why it should be philosophically absurd to assume this view. I cannot see the absurdity involved. For perfection means that body *cannot* be surpassed in the relevant sense. This goes beyond the statement that as a matter of fact there are only three dimensions. It is impossible that there should be something that is extended in more than three dimensions. Bodies are perfect magnitudes because they cannot be surpassed. And this is why Aristotle later says that lines and surfaces are deficient. Lines and surfaces can be surpassed. Thus, they are posterior to bodies.

[14] Cf. *Metaph.* V.16 1021b12–22a3. See also *Cael.* II.4 286b18–19, *Ph.* II.6 207a8–14, *Metaph.* X.4 1055a10–16. Cf. the discussion in Betegh et al. 2013, 44ff.

[15] See also Falcon 2005, 35.

[16] *Metaph.* V.16 1021b23–24.

[17] Wildberg 1988, 22.

Proposition 6. *Bodies are complete and perfect in the following way: Body is the only magnitude that is dimensionally complete because it is extended in all possible dimensions. In virtue of being extended in all possible dimensions body is prior to lower-dimensional magnitudes and it is not possible that there should be another magnitude that surpasses bodies.*

In the following section I will elaborate on the claim of priority. Body is prior to lower-dimensional magnitudes because it is extended in more dimensions than they are. This, I have argued, is a claim about the perfection of bodies. But there is an objection to this account. Speaking of *perfection* implies a certain normativity. But the modal claim that body is complete could be understood in a weaker sense. It could simply mean that body is three-dimensional and therefore is extended in more dimensions than, for example, a surface, and additionally that it is impossible that there is another magnitude with more than three dimensions. It may seem off the mark to speak of normativity here. If we understand 'normativity' in the sense of (morally) good or bad, this criticism is indeed justified. But this is not what perfection expresses. Perfection, as I understand it, is due to the systematic connection between dimensionality and substantiality. Extended substances are three-dimensional due to their substance and nature. At this point the third sense of *teleion* distinguished above becomes important. The normativity of three-dimensional extension is due to the fact that bodies can be seen as the endpoint or goal of a teleological quasi-process. This is the claim of the next section.

6.2.2. *Substantiality and the dimensions*

The connection between substantiality and the dimensions is not at all obvious. It is a topic that is rarely discussed either by Aristotle or by his commentators. Accordingly, it is difficult to reconstruct Aristotle's view on these matters. I offer here what I think is a possible and plausible interpretation. Other interpretations might be possible, too, and my interpretation is tentative. Having said that, the line of my argument runs thus: I will first argue that Aristotle does not conceive of the extension as a substance. This conclusion, I propose, can be drawn from the famous 'stripping' argument in *Metaphysics* VII.3. Being three-dimensional is not part of the definition of physical substances. Even though all physical substances are bodies, being a body, that is, being three-dimensionally extended, is not part of their essence.[18]

[18] As I said in Chapter 2, I disagree with Studtmann 2002 who thinks that body is a genus in the category of substance and a genus in the category of quantity. Because it is not part of the essence of a physical substance to be extended in three dimensions (though it is extended in three dimensions out of necessity),

However, even if extension in three dimensions is not the essence of a physical substance, it is connected to its essence. I will attempt to spell this out in terms of final causation. The three-dimensional extension of physical substances is due to and *flows from* their nature.

6.2.2.1. QUANTITY IS NOT A SUBSTANCE

In a first step I wish to argue that being extended is not part of the essence of a physical substance. If so, it cannot be a substantial feature or substance *of* a substance. In the course of the famous 'stripping' argument in *Metaphysics* VII.3 Aristotle remarks the following:[19]

> If it (i.e. matter) is not a substance, it is hard to see what else could be; for when all else is taken off, nothing apparent remains. For while other things are attributes, products, and capacities of bodies, length, breadth, and depth are quantities and not substances (for a quantity is not a substance). Rather, the substance is that primary thing to which these quantities belong. And yet when length, breadth, and depth are taken away, we see nothing left behind unless there be something which is determined by these, so on this view it must appear that matter alone is substance.[20] (*Metaph.* VII.3 1029a10–19)[21]

This passage occurs in a notoriously difficult and opaque discussion of substance.[22] One criterion for being a substance is, Aristotle says, not to be predicated of a subject, while other things are predicated of it.[23] In the course of this discussion Aristotle presents the 'stripping' argument. I am not going to join the debate on what 'matter' means here, or on whether the whole of the argument represents Aristotle's opinion, and whether this passage either affirms or denies prime matter (but see fn. 26 in this chapter). For our purposes the important question is how quantities and magnitudes occur in the course of the argument. This is to say, the focus lies on lines 12–16 where Aristotle apparently allows quantities to have a peculiar status.

For as it stands, the argument introduces an asymmetry between quantities and other properties. Roughly, the asymmetry is that other properties, like colours, are

there is, according to my interpretation, no 'body problem' in Aristotle. For the body problem to arise it is not sufficient that physical substances are bodies, even necessarily so. They must have part of their essence in common with body in the category of quantity. But, as Studtmann 2002, 215 himself notes, Aristotle never makes this claim. And I will argue that Aristotle has good reasons for not making this claim.

[19] I want to thank Alan Code for pointing out the importance of this passage for the present context and discussing it with me.

[20] *εἰ γὰρ μὴ αὕτη οὐσία, τίς ἐστιν ἄλλη διαφεύγει· περιαιρουμένων γὰρ τῶν ἄλλων οὐ φαίνεται οὐδὲν ὑπομένον· τὰ μὲν γὰρ ἄλλα τῶν σωμάτων πάθη καὶ ποιήματα καὶ δυνάμεις, τὸ δὲ μῆκος καὶ πλάτος καὶ βάθος ποσότητές τινες ἀλλ' οὐκ οὐσίαι (τὸ γὰρ ποσὸν οὐκ οὐσία), ἀλλὰ μᾶλλον ᾧ ὑπάρχει ταῦτα πρώτῳ, ἐκεῖνό ἐστιν οὐσία. ἀλλὰ μὴν ἀφαιρουμένου μήκους καὶ πλάτους καὶ βάθους οὐδὲν ὁρῶμεν ὑπολειπόμενον, πλὴν εἴ τί ἐστι τὸ ὁριζόμενον ὑπὸ τούτων, ὥστε τὴν ὕλην ἀνάγκη φαίνεσθαι μόνην οὐσίαν οὕτω σκοπουμένοις.*

[21] Translation is by Morison 2002, 111. The translation is a modification of Bostock 1994.

[22] The literature on this passage is vast. A first orientation can be provided by the commentaries of Bostock 1994; Frede and Patzig 1988; Detel 2009. For two recent interpretations see Green 2014; Lewis 2013.

[23] *Metaph.* VII.3 1029a7–8.

predicated of quantities, but not vice versa. The subject of colour are surfaces, but the subject of surfaces are not colours. In this sense, magnitudes underlie other properties.[24] Because of this asymmetry one might believe—and some philosophers, presumably Platonists, have in fact believed[25]—that quantities, like mathematical bodies, surfaces, lines, or points, are substances. One might believe this because if substance is characterized by an is-said-of relation it seems that all other properties are said of magnitudes, but magnitudes are not said of them. Thus, in stripping properties from an object one arrives at some point at the idea of pure quantities. That is to say, there seems to be a stage where we get to objects whose only essential properties are quantitative ones.

However, Aristotle is explicit in stating that magnitudes are not substances. For he immediately adds that the length, breadth, or depth of a thing is not its substance. Even if in the course of this 'stripping' process we reach magnitudes after all other properties are taken away, the magnitudes themselves must belong to something. And, as Aristotle continues, matter—and if matter qualifies as substance, the substance—is that to which length, breadth, and depth belong. If we take this idea seriously, we see that three-dimensional extension cannot be, wholly or partly, the essence of a substance.[26]

[24] Cf. *Metaph.* III.5 1001b32–1002a4; *Metaph.* V.18 1022a16–17. The same point is made by Morison 2002, 110.

[25] Cf. *Metaph.* III.5 and XIII.2.

[26] The passage, of course, is less famous for what it implies about the metaphysical status of quantities, than for what it implies for the metaphysical status of substance and matter. It is, alongside *GC* II.1, the main passage that is discussed under the heading 'Did Aristotle believe in prime matter?' Though Aristotle's theory of substance or matter is not our topic, there is a certain complication we must address in this footnote. For I have just argued that Aristotle believes that extension (in whatever dimension) is not a substance. However, to say that the extension of a body is not its substance contradicts the view of those who believe that extension *is* prime matter. The controversy revolves around the immediately following sentence: 'And yet when length, breadth, and depth are taken away, we see nothing left behind unless there be something which is determined by these, so on this view it must appear that matter alone is substance' (*Metaph.* VII.3 1029a16–19). The sentence can be understood in several ways. First, it might mean that a determinate length, breadth, or depth is taken away. According to this interpretation, if I take away the length of a line, I take away the specific length of the line. If the line is one metre long, I take away the length of one metre. What I am left with is, however, still a one-dimensional extension, but an indefinite one. It is extended, but not in a determinate length. This interpretation, which ultimately goes back to Simplicius, is put forward, e.g. by Sorabji 1985. Second, it might mean that by taking away length, breadth, and depth extension in one, two, or three dimensions is taken away. What is left, according to the second interpretation, is something that is not extended in any dimensions. This interpretation goes back to Philoponus, but has been developed in different directions. Schofield 1972, e.g. believes that literally nothing remains. The problem with this is, as Bostock 1994, 77 remarks, that it becomes a mystery why Aristotle should think that *something*, i.e. matter, remains. Third, one could also suppose that what remains is ordinary matter, like, for example, bronze. It is the bronze that is bounded by length, breadth, and depth. Cf. Morison 2002, 111.

In the present context only the first of these alternatives is important, since it is the only interpretation that identifies extension with matter. I think that this interpretation is not correct. I cannot go into a detailed criticism, but simply note some points that speak against Sorabji's interpretation. First, in Aristotle's regular use, the words 'length, breadth, and depth' designate one-, two-, or three-dimensional extension in general and not a specific size of an object. Cf. *Metaph.* V.13 1020a11–12. Thus, to take away length, breadth, and depth is to take the dimensions of an object and not only its specific size. Second, it is quite plausible that the

It is a feature of physical substances, but it is not part of their essence. Humans, dogs, or cats do not have three-dimensional extension as part of their essence. Essentially they are something else, namely living beings.

On the other hand, physical substances are not extended in three dimensions by chance. It is predicated of them *qua* being physical substances:[27]

Proposition 7. *x is a physical substance if and only if x is extended in three dimensions.*[28]

Proposition 8. *If x is a physical substance, being extended in three dimensions is not part of the essence of x.*

These two propositions give us a necessary and sufficient connection between being a physical substance and being extended in three dimensions, but they deny that it is part of the essence of physical substances to be extended in three dimensions. However, it seems to me a plausible assumption that it is not entirely detached from their essence either. It is not a brute fact that these two are found in constant conjunction. There is a connection between the essence of a physical substance and its being a body. To use a metaphor, it *flows* from its essence that a physical substance

passage is directed against a Platonic view. This would explain the obvious parallel to *Metaph.* III.5. In this case the ultimate aim of the passage would be a *reductio* of a rival theory, not a presentation of Aristotle's own theory. Third, the argument is entirely based on the logic of predication, as Aristotle himself emphasizes. The account of predication that Aristotle uses comes from the *Organon*. There being a subject is identified with being a substance by way of the is-said-of relation. But one should not suppose that one can, as it were, read off the metaphysical status of objects by focusing solely on that criterion. From the standpoint of predication, quantities underlie other properties. But it is not an ontological point because a physical substance is not *made out* of a quantity or extension. Thus, insofar as the concept of matter is connected to that out of which something is made of (*Metaph.* IX.7), this passage has nothing to contribute. Matter is defined by its potential to be the substance, not by its being a quantity of stuff (*Metaph.* VIII.1 1042a27–28; *GC* I.3 317b16–18). Even if Sorabji were correct and indefinite extension remained, I can see no reason to assume that indefinite extension plays a role analogous to the role of bronze in a brazen statue.

On this account I also disagree with Studtmann 2006 who argues that extension 'exists in material composites as the matter for substantial forms' (Studtmann 2006, 182). Extension is not *literally* the matter. Extension is a *feature* of the matter. Studtmann thinks that in studying physical or mobile bodies one studies the matter of material substances; I believe that one studies body (a quantity) insofar as it belongs to physical substances. For further discussion of my view on extension see Section 6.4.

[27] For an interpretation of the *qua*-locution see Section 4.1.1.1 and the literature mentioned there.

[28] One may object that this proposition is false because mathematical bodies are extended in three dimensions, but are not physical substances. But one should note that this proposition tells us nothing about the respective definitions, but makes an existence claim. And for Aristotle it is true, I assume, that there are no three-dimensional extended objects which exist independently of physical substances (with the possible exception of the whole cosmos whose status as a substance is unclear. But it is clearly a physical object. See Matthen and Hankinson 1993 for discussion). Yet, since the focus of this section is on Proposition 8 as well as on the question of how three-dimensionality flows from the essence of physical substances, we may also use the weaker proposition

Proposition 7*. x is a physical substance only if x is extended in three dimensions.

Those who are in serious doubt about Proposition 7 can read the starred version instead.

is extended in three dimensions.[29] Aristotle, I propose, explains this connection between three-dimensionality and the essence by a comparison to a quasi-natural process.

6.2.2.2. THREE-DIMENSIONALITY AND PRIORITY IN SUBSTANCE

Though three-dimensionality is not the essence of a physical substance, it is, I will argue, tied to the essence of it. It is not a constituent, but a concomitant of the essence of physical substances. Being three-dimensional follows from the essence of physical substances. Aristotle shows this by relying on the principle that what is prior in generation is later in substance.[30] Bodies represent the endpoint of a quasi-natural process. That is to say, there is a quasi-generation from lines and surfaces to bodies, which, being the end point of such a process, become alive. Three-dimensionality is prior in nature to lower-dimensional entities because it can be seen as the result of a quasi-natural process of generation and is tied to the concept of a living substance, or, more generally, to the concept of the nature of a thing.

> Again, the generations show that we are right. First it comes to be in length, then breadth, lastly depth, and it is complete. If, then, that which is posterior in generation is prior in substance, the body should be prior to plane and length. It is more complete and whole in the following way also—it becomes animate. How, on the other hand, could there be an animate line or a plane? The supposition passes the power of our senses.[31] (*Metaph.* XIII.2 1077a24–31)[32]

The context of the passage is an argument against the claim that lines, planes, and surfaces are prior to perceptible substances. Aristotle considers two considerations against this claim. First, it is a general truth that what is prior in generation is later in substance. Since lines and surfaces are prior to bodies in generation, they are, according to this principle, posterior in substance. Second, a body becomes animate. All living things are three-dimensional, but it is impossible to see how there could be a living surface.

This argument has puzzled commentators. And it is indeed puzzling that Aristotle should assert that there is a generation of bodies from lines. Julia Annas, for example,

[29] In regard to this metaphor, it is interesting to note that some scholastic authors took such metaphors quite seriously and argued that substances bring about their attributes in an efficient causal way. Suárez, e.g. argues that 'accidental properties, especially those that follow upon or are owed [to a substance] by reason of its form, are caused by the substance not only as a material cause or final cause, but also as an efficient cause through a natural resulting ([nimirum] proprietates accidentales, praesertim illas quae consequuntur aut debentur rei ratione formae, causari a substantia non solum materialiter et finaliter, sed etiam effective per naturalem resultantiam)' (DM 18.3.4 [25, 616a]). The thought I will ascribe to Aristotle is, however, closer to final causation.

[30] Cf. *Metaph.* IX.8 1050a4–10, *Ph.* VIII.7 261a13–21, *GA* II.6 742a18–22.

[31] ἔτι αἱ γενέσεις δηλοῦσιν. πρῶτον μὲν γὰρ ἐπὶ μῆκος γίγνεται, εἶτα ἐπὶ πλάτος, τελευταῖον δ' εἰς βάθος, καὶ τέλος ἔσχεν. εἰ οὖν τὸ τῇ γενέσει ὕστερον τῇ οὐσίᾳ πρότερον, τὸ σῶμα πρότερον ἂν εἴη ἐπιπέδου καὶ μήκους· καὶ ταύτῃ καὶ τέλειον καὶ ὅλον μᾶλλον, ὅτι ἔμψυχον γίγνεται· γραμμὴ δὲ ἔμψυχος ἢ ἐπίπεδον πῶς ἂν εἴη; ὑπὲρ γὰρ τὰς αἰσθήσεις τὰς ἡμετέρας ἂν εἴη τὸ ἀξίωμα.

[32] Translation adapted from Annas 1976, 94.

thinks that there is a confusion on Aristotle's part. Aristotle, she believes, fails to distinguish between mathematical and physical objects.[33] Additionally, one might object that the term 'generation' is used in a homonymous sense. For though it might be true that in the generation of physical bodies what is prior in being comes later in generation, this isn't the case with mathematical entities, where 'generation' must mean 'construction'. There is no generation, no process that leads from surfaces to three-dimensional bodies. The man is prior to the boy in substance, but the boy is prior in generation to the man, as Aristotle explains in *Metaph.* IX.8 1050a5. And we may agree with Aristotle that this is the case. But whether or not we agree, our agreement would be conditional on the fact that there is a natural process leading from the boy to the man. Since there is no natural process from lines to bodies, why should we believe that bodies are prior in nature? Aristotle's argument, we may say, is simply irrelevant.

> This ambiguity [between the two senses of generation] deprives the argument of whatever value it might otherwise have possessed. (Ross 1924a, 414)

And Ross seems right. It seems all too easy to charge Aristotle with being confused and to treat the passage as an isolated and in the final analysis unintelligible piece of writing.

I think that these objections are not entirely justified. First, the claim that there is a transition from lower-dimensional to higher-dimensional magnitudes is not restricted to *Metaphysics* XIII.2, but appears in the *De Caelo* as well. Second, the context of both passages is dialectical.[34] That is to say, the key element of transition between extended objects is a common presupposition shared by Aristotle's opponents.

Beginning with the first remark, the view that there is a transition is also endorsed in the previously quoted passage from the first chapter of *De Caelo*.[35]

> One thing, however, is clear. There is no transition to another kind of magnitude, as we passed from length to surface, and from surface to body. For if we could, it would cease to be true that body is complete magnitude. We could pass beyond it only in virtue of a defect in it; and that which is complete cannot be defective, since it is in all ways. (*Cael.* I.1 268a30–b5)

When one closely reads the passage, it becomes obvious that Aristotle not only denies that there is a transition from body to another genus, but he apparently assumes that there *is* a transition from line to surface and surface to body. This obviously connects this passage to the passage in *Metaphysics* XIII.2. Moreover, the choice of language betrays that the transition Aristotle has in mind is not a logical construction, but rather a quasi-natural generation. The word *ekbasis* occurs only here in the *corpus aristotelicum* and *metabasis* usually describes the elemental transformations.[36]

[33] Cf. Annas 1976, 146. [34] Cf. Cleary 1995; Betegh et al. 2013. [35] See p. 79.
[36] Cf. *Cael.* III.1 298b1, III.7 306a32. See also Bonitz 1870 and the TLG.

Hence, in their usual meaning these words describe a physical transition. This makes it unlikely that Aristotle wants to restrict the processes referred to here to a mathematical transition.[37] In this sense, the passage from *Metaphysics* XIII.2 cannot be treated as an isolated passage. This is especially apparent when we consider that the conclusion that there is no further transition is central to Aristotle's own theory. Aristotle endorses the conclusion.

This, however, does not mean that the conclusion cannot be situated in a dialectical context. Aristotle endorses the conclusion, but his premisses may partly come from a dialectical context. What could this context be? What could Aristotle have in mind here? One possible candidate is a passage in Plato's *Laws*:

> What happens when the generation of all things occur? Clearly, an *archê* takes up growth, and reaches a second stage and then the next one out of this second, so that as soon as it reaches the third, there is something for percipient things to perceive.[38] (Pl. *Lg.* X 894a1–5)

This passage is crucial for my interpretation in several respects.[39] First, *metabasis* refers here as well as in *De Caelo* I.1 and *Metaphysics* XIII.2 to a transition from n dimension to n+1 dimension. Second, the phrase 'there is something for percipient things to perceive' unambiguously shows that it is a generation of physical, perceptible bodies. Finally, the parallel with *Laws* X also shows that Aristotle is referring here to a doctrine which he may or may not endorse, but which certainly does not originate with him. Rather it is a presupposition Aristotle's opponents subscribe to and, therefore, Aristotle is justified in drawing on this presupposition in the context of an argument against his opponents.

The dialectical context, I propose, can be described as follows: anyone who believes that there is a transition between magnitudes that finally leads to physical substances must at least agree that the transition stops at the third dimension. If so, one must further agree that bodies are complete and perfect. The process of a generation is completed when three-dimensional magnitudes are reached. Since, and this is Aristotle's own premiss, what is prior in generation is later in substance, bodies are prior in substance to two-dimensional objects.

However, once we have reached this conclusion, we could, in a modification of Wittgenstein's famous saying, throw away the ladder, after we have climbed up on it.[40] That is to say, we can draw the conclusion even if we deny that there literally is a transition from n to n+1 dimensions. We can establish the conclusion that bodies are prior in substance to lower-dimensional magnitudes by relying on the

[37] This conclusion is reinforced by the fact that the context is physical science.
[38] *γίγνεται δὴ πάντων γένεσις, ἡνίκ' ἂν τί πάθος ᾖ; δῆλον ὡς ὁπόταν ἀρχὴ λαβοῦσα αὔξην εἰς τὴν δευτέραν ἔλθῃ μετάβασιν καὶ ἀπὸ ταύτης εἰς τὴν πλησίον, καὶ μέχρι τριῶν ἐλθοῦσα αἴσθησιν σχῇ τοῖς αἰσθανομένοις.*
[39] On this passage see also Betegh et al. 2013.
[40] The original says: 'He must so to speak throw away the ladder, after he has climbed up on it' (Wittgenstein 1961, 6.54). We, on the other hand, need not throw away the ladder, although we are free to do so.

model of natural generation. For Aristotle this model may not be literally true, since there is no transition between the dimensions. But even if not literally true, this model provides an insight into the relations of priority between dimensional objects. In other words, Aristotle can claim that in the case of magnitudes an analogue to the priority in substance and generation is true. The priority of body over lower-dimensional magnitudes can be made transparent, if we compare it to the case of a natural generation. This comparison is especially apt since Aristotle's opponents in fact believe that there is a natural generation.

This interpretation faces the following objection.[41] If we detach the dialectical context, the conclusion does not follow. If there is no generation, then we cannot apply the rule that what is posterior in generation is prior in substance. You cannot climb a false ladder, so to speak. This objection has a point. Surely, one cannot establish a conclusion by means of a false premiss. But this is not the way I would construe the argument. As I said, Aristotle uses the priority in nature as an analogue. This analogue is especially fitting since some philosophers, presumably Platonists, in fact believe in generation of planes from lines. The analogue works insofar as it gives a suitable reason to assume that three-dimensional objects are perfect. This reason is, of course, defeasible and Aristotle has not given sufficient support for *proving* the claim. But it makes the claim more credible. It shows that his opponents are committed to this conclusion by their own standards. It also provides, through the analogue of priority in substance, a way to understand what it means to claim that bodies are perfect.

6.2.2.3. SUBSTANCE, CAUSES, AND DIMENSIONALITY

My claim is that we should interpret Aristotle's remarks in the following way: Though Aristotle did not believe that there is literally a transition from n-1 to n-dimensions, he did believe that bodies are complete and perfect. He engages in a dialectical argument that presupposes that there is a transition from n-1 to n-dimension. If one accepts that there is a transition, one has to admit that there cannot be more than three dimensions. But even if one does not believe that there is a transition, one can establish a link between the essence of a physical substance and its being three-dimensional. Aristotle alludes to this link when he says that only bodies become animate. All extended living substances are three-dimensional bodies.[42]

This, however, is not a mere coincidence. Substances are three-dimensional *in virtue* of their essence. This is, I suggest, the crucial connection between the idea of a natural process of generation depicted in *Metaphysics* XIII.2 and *De Caelo* I.1 and our question how dimensionality and essence connect. Bodies are perfect and complete because they are three-dimensional. The priority of three-dimensionality, on the other hand, cannot be established without taking into account the *nature* of physical substances. The argument for the priority of bodies is grounded in considerations about the nature of physical substances to which the bodies belong.

Proposition 9. *If x is a physical substance, x is a three-dimensional body in virtue of its essence and nature.*

As I said earlier, this line of interpretation is not without its problems. It is difficult to explain the notion of priority in nature with regard to magnitudes once the assumption of a literal transition between objects of different dimensionality is given up. I chose to employ the notions of 'being due to' or 'in virtue of'. The thought is that, although three-dimensionality is not part of the essence of physical substances, it is still connected to their essence. The essence or nature of physical substances is responsible for their being three-dimensional. Of course, to a certain extent these notions are metaphors. But I believe that sense can be made of those metaphors. This also provides the background for the discussion of the next section where the status of limits will be discussed. Since bodies are complete and ontologically prior to lower-dimensional items, the being of those items is grounded in the being of bodies.

6.3. Bodies and Limits

One intriguing topic in any discussion—be it modern or ancient—of extended objects is the nature of limits. How should we think of limits? Consider the case of a body and the surface that is its boundary. Intuitively, the world contains surfaces. A golden sphere has a surface as its limit. But what is this surface? There seem to be two radically distinct ways to think of this surface. Recently, Galton has argued that the existence of these two ways leads to what he calls the 'paradox of surfaces' (Galton 2007, 379).[43] Galton argues that one way to think of the surface of a golden sphere is as made of gold.[44] If you scratch a golden sphere at its surface, you scratch a golden surface. The surface is golden. It is a surface that is made of gold. On the other hand, it belongs to our concept of a surface that a surface is two-dimensional. But you cannot scratch something two-dimensional, nor can anything two-dimensional be made of gold. Moreover, if we accept the second conception of a surface, more questions come up. Shall we say that there are really two-dimensional layers in the world? Or shall we treat them as abstractions, as Whitehead did? If we accept two-dimensional layers in our ontology, do they belong to the objects in question? Obviously, this introduces a whole battery of problems concerning open and closed bodies. Aristotle, no less than other

[41] This was suggested to me by Benjamin Morison.

[42] Of course, Plato and Aristotle believe that there are non-extended living beings, e.g. the demiurge in Plato's *Timaeus* or the unmoved mover in Aristotle's *Metaphysics*. The point is that among extended things (including points) only three-dimensional entities can be substances.

[43] The two distinct ways of conceptualizing a surface are already mentioned by Stroll 1999, who calls them the 'Somorjai' and the 'Leonardo' conception.

[44] Galton 2007, 379.