Automata and Grammars

SS 2018

Assignment 13

Solutions are to be presented at the **Seminary** on **Thursday, May 24, 2018**.

Problem 13.1. [Encodings of Turing Machines]

Let $M = (\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \{0, 1, \Box\}, \Box, q_0, q_3)$ be the Turing machine that is given through the following transition function:

δ	0	1	$\Box$	Comments
q_0	$(q_0, 0, R)$	$(q_0, 1, R)$	$(q_2, \Box, L)$	Move right to the last nonblank symbol
q_1	$(q_1, 0, L)$	$(q_1, 1, L)$	$(q_3, \Box, R)$	Move left to the first nonblank symbol
q_2	$(q_1, 1, L)$	$(q_2, 0, L)$	$(q_3, 1, 0)$	Add 1 while moving left
q_3	$-$	$-$	$-$	Final state

that is, M computes the binary +1-function (compare M to the TM in the first example of Section 4.1).

(a) Compute the encoding $c(M)$ of the above TM (see the definition before Lemma 4.9).

(b) Let M' be the TM that is given through the following encoding

$$\begin{aligned} c(M') \;=\; & 1110001111101010100100011010010101000110100010010101 1\\ & 001010010010110010010010101100100010001001001 11. \end{aligned}$$

Reconstruct the TM $M' = (Q, \Sigma, \Gamma, \Box, \delta', q_0, q_n)$ from its encoding $c(M')$.

(c) Construct the encoding $c(\hat{M})$ of the following TM $\hat{M}$ from the encoding $c(M')$ of M' and the input word $x = 10$, where $\hat{M}$ behaves as follows:

(1) erase the given input;
(2) write x;
(3) simulate M' on input x.

Problem 13.2. [Recursively Enumerable Languages]

Prove that the following languages are recursively enumerable:

(a) $L_1 = \{ w \in \{a, b, c\}^* \mid |w|_a = |w|_b + |w|_c \}$,
(b) $L_2 = \{ w \in \{a\}^* \mid \exists n \geq 0 : w = a^{2^n} \}$.

Problem 13.3. [Undecidable Languages]

Prove that the following languages are undecidable:

(a) $H_0 = \{ w \in \{0, 1\}^* \mid \text{The TM } M_w \text{ halts on empty input} \}$,
(b) $H_\forall = \{ w \in \{0, 1\}^* \mid \text{The TM } M_w \text{ halts for every input} \}$,
(c) $T_2 = \{ u\#v \mid u, v \in \{0, 1\}^* \text{ and the TM } M_u \text{ halts for all inputs for which the TM } M_v \text{ halts} \}$.

Problem 13.4 [Non-Recursively Enumerable Languages]

Prove that the following languages are not even recursively enumerable:

(a) $L_1 = \{ w \in \{0,1\}^* \mid \text{The TM } M_w \text{ does not halt on empty input} \}$,
(b) $L_2 = \{ w \in \{0,1\}^* \mid \text{The TM } M_w \text{ does not halt on any input} \}$,
(c) $L_3 = \{ u\#v \mid u, v \in \{0,1\}^* \text{ and the TMs } M_u \text{ and } M_v \text{ halt on the same inputs} \}$.