Automata and Grammars

SS 2018

Assignment 12

Solutions are to be presented at the **Seminary** on **Thursday, May 17, 2018**.

Problem 12.1. [Turing Machines]

Design a one-tape Turing machine M_1 with at most 8 states such that

$$L(M) = \{\, a^n b^n c^n \mid n \geq 0 \,\}.$$

Problem 12.2. [Turing Machines]

Let $L_{\text{copy}} = \{\, wcw \mid w \in \{a, b\}^* \,\}$.

(a) Design a one-tape Turing machine M_1 with at most 8 states such that $L(M_1) = L_{\text{copy}}$.

(b) Design a two-tape Turing machine M_2 with at most 4 states such that $L(M_2) = L_{\text{copy}}$.

Problem 12.3. [Turing Machines]

Let $f : \mathbb{N} \to \mathbb{N}$ be the function $f(n) = |\text{dya}(n)|_2$, that is, for each non-negative integer n, $f(n)$ is the number of occurrences of the digit 2 in the dyadic representation of n. Construct a two-tape Turing machine M with at most 8 states that computes the function f.

Hint: The dyadic representation of a positive integer n is the word $w = a_m a_{m-1} \cdots a_1 a_0 \in \{1, 2\}^+$ such that $n = \sum_{i=0}^{m} a_i \cdot 2^i$. The advantage of the dyadic representation over the binary representation is the fact that it establishes a bijection between the set of positive integers and the set of words $\{1, 2\}^+$, while the binary representation is not unique if leading zeros are allowed.

Problem 12.4 [Phrase-Structure Grammars]

Determine the languages that are generated by the following general grammars:

(a) $G_1 = (\{S, A, B, C, D, E\}, \{a, b\}, P_1, S)$, where P_1 is defined as follows:
$P_1 = \{S \to EC, S \to \varepsilon, C \to ACa, C \to BCb, C \to D,$
$aA \to Aa, bA \to Ab, aB \to Ba, bB \to Bb, EA \to Ea, EB \to Eb,$
$aD \to Da, bD \to Db, ED \to \varepsilon\}$,

(b) $G_2 = (\{S, A, B\}, \{a, b\}, P_2, S)$, where P_2 is defined as follows:
$P_2 = \{S \to ASB, S \to BSA, S \to SS, S \to \varepsilon,$
$AB \to \varepsilon, BA \to \varepsilon, A \to a, B \to b\}$.