Automata and Grammars

SS 2018

Assignment 11

Solutions are to be presented at the **Seminary** on **Thursday, May 10, 2018**.

Problem 11.1. [Emptiness and Finiteness]

Determine the cardinality of the language $L(G_i)$ for the following context-free grammars G_i $(i = 1, 2, 3)$:

(a) $G_1 = (\{S, A, B, C, D\}, \{a, b\}, P_1, S)$, where P_1 is defined as follows:

$$\begin{aligned} P_1 &= \{S \to aS, S \to AB, S \to CD, A \to aDb, A \to AD, A \to BC, \\ &\quad B \to bSb, B \to BB, C \to BA, C \to ASb, D \to ABCD, D \to \varepsilon\}. \end{aligned}$$

(b) $G_2 = (\{S, A, B, C, D\}, \{a, b\}, P_2, S)$, where P_2 is defined as follows:

$$\begin{aligned} P_2 &= \{S \to AB, S \to BC, S \to CD, A \to BC, A \to BD, \\ &\quad B \to BC, B \to DD, B \to b, C \to AC, C \to BC, D \to a\}. \end{aligned}$$

(c) $G_3 = (\{S, A, B, C, D\}, \{a, b\}, P_3, S)$, where P_3 is defined as follows:

$$\begin{aligned} P_3 &= \{S \to AB, S \to BC, S \to CD, A \to BC, A \to BD, \\ &\quad B \to CC, B \to DD, B \to b, C \to AS, D \to AC, D \to a\}. \end{aligned}$$

Problem 11.2. [CKY-Algorithm]

Apply the CYK-algorithm from the proof of Theorem 3.30 to the following context-free grammar

$$G = (\{A, B, C, R, S, T, U, V, X, Y, Z\}, \{a, b, c\}, P, S)$$

and the input words $w_1 = bcaaacbb$ and $w_2 = abcbaabc$, where P is defined as follows:

$$\begin{aligned} P_1 &= \{S \to TU, T \to BA, T \to BX, T \to BY, X \to TA, \\ &\quad Y \to CA, Y \to RA, U \to AB, U \to AV, U \to AZ, \\ &\quad Z \to UB, V \to RB, R \to CR, R \to c, A \to a, B \to b, C \to c\}. \end{aligned}$$

Problem 11.3. [DPDA]

In Theorem 3.17 we have seen that, for each PDA M_1, there exists a PDA M_2 such that $N(M_2) = L(M_1)$, while for deterministic PDAs, a corresponding result does not hold. Why doesn't the proof of Theorem 3.17 carry over to DPDAs?

Problem 11.4. [DPDA]

Consider the DPDA $M = (\{q_0, q_1, q_2\}, \{a, b\}, \{\#, Z\}, \delta, q_0, \#, \{q_2\})$, where the transition function δ is defined as follows:

$$\begin{array}{rcl} \delta(q_0, a, \#) & = & (q_1, \#Z), \\ \delta(q_1, a, Z) & = & (q_1, ZZ), \\ \delta(q_1, b, Z) & = & (q_2, \varepsilon), \\ \delta(q_2, b, Z) & = & (q_2, \varepsilon). \end{array}$$

(a) Give a nonempty input word $w \in \{a, b\}^*$ that M does not read completely.

(b) Use the construction from the proof of Lemma 3.33 to extend M to an equivalent DPDA M' that always reads its input words completely.

Problem 11.5. [Ogden's Lemma for DCFL]

Prove that the language $L = \{ a^n b^n c, a^n b^{2n} d \mid n \geq 0 \}$ is not deterministic context-free by applying Ogden's Lemma for DCFL (Theorem 3.40) to L.