Automata and Grammars

SS 2018

Assignment 10

Solutions are to be presented at the **Seminary** on **Thursday, May 3, 2018**.

Problem 10.1. [Context-Free Grammars to PDAs]

Let $G = (\{S, A, B, C\}, \{a, b, c\}, P, S)$ be the context-free grammar (in Greibach normal form) that is given by the following set of productions:

$$\begin{aligned} P \;=\; & \{S \to aABC, S \to bBCA, A \to aAA, A \to a, \\ & B \to bCB, B \to bB, B \to b, C \to cBC, C \to cC, C \to c\}. \end{aligned}$$

(1) Give a leftmost derivation for the word $w = bbccbcaaa$.

(2) Convert the grammar G_1 into a PDA M_1 using the construction from the proof of Theorem 3.19.

(3) Give the detailed computation of M_1 on input $w = bbccbcaaa$ that corresponds to the leftmost derivation from (1).

Problem 10.2. [PDA to Context-Free Grammar]

Let $M = (\{q_0, q_1\}, \{a, b\}, \{\#, Z\}, \delta, q_0, \#, \emptyset)$ be the PDA that is defined by the following transition relation:

$$\begin{array}{lll} \delta(q_0, a, \#) = \{(q_1, \#Z)\}, & \delta(q_0, \varepsilon, Z) = \{(q_0, \varepsilon)\}, & \delta(q_0, b, \#) = \{(q_1, \varepsilon)\}, \\ \delta(q_1, a, Z) = \{(q_1, ZZ)\}, & \delta(q_1, b, Z) = \{(q_1, Z), (q_0, \varepsilon)\}. & \end{array}$$

(1) Give an accepting computation of M for the input $w = aabbabb$.

(2) Convert the PDA M into a context-free grammar G using the construction from the proof of Theorem 3.20.

(3) Give a leftmost derivation for the word $w = aabbabb$ in G that corresponds to the computation from (1).

Problem 10.3. [Shuffle Operation]

The operation $\sqcup$ (shuffle) on Σ^* is defined as follows:

$$\forall\, u, v \in \Sigma^* : u \sqcup v := \{\, u_1 v_1 u_2 v_2 \cdots u_n v_n \mid n \geq 1, u = u_1 u_2 \cdots u_n \text{ and } v = v_1 v_2 \cdots v_n \,\}.$$

This operation is extended to sets of words (languages) as follows, where $L_1, L_2 \subseteq \Sigma^*$:

$$L_1 \sqcup L_2 = \bigcup_{u \in L_1, v \in L_2} (u \sqcup v).$$

(a) Determine the set $a^2 b^2 \sqcup c^2$.

(b) Determine the set $L_1 \sqcup L_2$ for $L_1 = \{\, a^m b^m \mid m \geq 0\}$ and $L_2 = \{\, c^n \mid n \geq 0 \,\}$.

(c) Prove that $L_1 \sqcup L_2$ is a context-free language for each regular language L_1 and each context-free language L_2.