Automata and Grammars

SS 2018

Assignment 9

Solutions are to be presented at the **Seminary** on **Thursday, April 26, 2018**.

Problem 9.1. [Greibach Normal Form]

Convert the context-free grammar $G_1 = (\{S, E, F\}, \{a, (,), +, *\}, P_1, S)$ into an equivalent grammar that is in Greibach Normal Form, where

$$P_1 = \{S \to (E), E \to F + F, E \to F * F, F \to a, F \to S\}.$$

Use the construction detailed in the proof of Theorem 3.10, and notice that this construction already applies to context-free grammars that are in weak CNF.

Problem 9.2. [The Pumping Lemma for Context-Free Languages]

Prove that the following languages are not context-free by applying the Pumping Lemma for context-free languages (Theorem 3.14):

(a) $L_1 = \{ a^i b^i c^{2i} \mid i \geq 1 \}$,

(b) $L_2 = \{ a^i b^j c^k \mid 0 \leq i \leq j \leq k \}$,

(c) $L_3 = \{ ww \mid w \in \{a, b\}^* \}$.

Problem 9.3. [Context-Free Languages]

Determine which of the following languages are context-free:

(a) $L_1 = \{ a^m b^n \mid 0 \leq m \leq n \leq 2m \}$,

(b) $L_2 = \{ w \in \{a, b, c\}^* \mid |w|_a = |w|_b = |w|_c \}$,

(c) $L_3 = \{ a^{n_1} b a^{n_2} b a^{n_3} \mid n_1 \geq n_2 \geq n_3 \geq 0 \}$.

Hint: You are expected to provide proofs for your answers!

Problem 9.4. [Pushdown Automata]

Give an example of an accepting computation for the following PDA

$$M = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \{A, \#\}, \delta, q_0, \#, \{q_3\}),$$

where δ is given by

δ	q_0	q_1	q_2	q_3
$(a, \#)$	$(q_1, \#A)$	–	–	–
(a, A)	–	(q_1, AA)	–	–
$(b, \#)$	–	–	–	–
(b, A)	–	(q_2, A)	(q_3, ε)	(q_2, A)

and determine the languages that are accepted by it by considering both acceptance conditions:

(1) acceptance by final state and

(2) acceptance by empty pushdown.

Problem 9.5. [Pushdown Automata]

Prove Theorem 3.18, that is, from a given PDA M_2, construct a PDA M_1 such that $L(M_1) = N(M_2)$.