Automata and Grammars

SS 2018

Assignment 8

Solutions are to be presented at the **Seminary** on **Thursday, April 19, 2018**.

Problem 8.1. [Context-Free Grammars]

Let $G = (\{S, A, B, C\}, \{a, b, c\}, P, S)$ be the context-free grammar with the following set of productions:

$$P = \{S \to aSb, S \to aAbb, S \to \varepsilon, A \to aAB, A \to bB, B \to aAb, B \to CC, C \to ba, C \to cS\}.$$

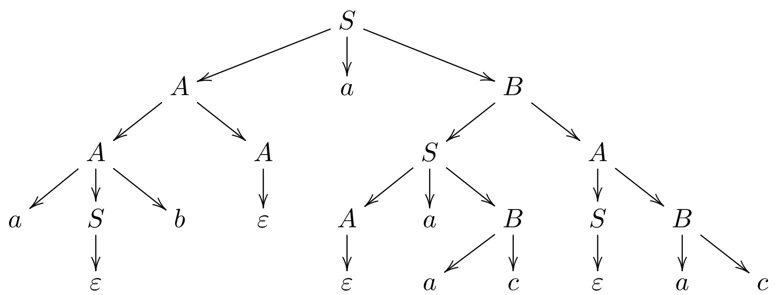
(a) Give a leftmost derivation for the word $w = aaababbababbababbb$.

(b) Give a right derivation for the word $w = aabbababacbb$.

(c) Give syntax trees for your derivations in (a) and in (b).

Problem 8.2. [Context-Free Grammars]

The following figure shows a syntax tree for some context-free grammar G and a word w:



(a) Determine the word w generated by this syntax tree.

(b) Give a leftmost derivation for this word.

(c) Give a list of all productions that are used in this syntax tree.

(d) What can be said about the ambiguity of the underlying grammar?

Problem 8.3. [Context-Free Grammars]

Provide context-free grammars for the following languages:

(a) $L_1 = \{ wcw^R \mid w \in \{a, b\}^* \}$,

(b) $L_2 = \{ a^k b^m c^n \mid k = m \text{ or } m = n \}$,

(c) $L_3 = \{ uv \mid u, v \in \{a, b\}^+, |u| = |v|, \text{ but } u \neq v \}$.

Problem 8.4. [Simplifying Context-Free Grammars]

Let G be the following context-free grammar:

$$G = (\{S, A, B, C\}, \{a, c\}, \{S \to aACa, A \to B, A \to a, B \to C, B \to c, C \to cC, C \to \varepsilon\}, S).$$

(a) Determine a proper context-free grammar that is equivalent to G.

(b) Remove the ε-productions from G.

(c) Remove the chain productions from G. Is the resulting grammar G' proper? If not, then determine a proper context-free grammar that is equivalent to G' and that has neither ε-productions nor chain productions.

Problem 8.5. [Chomsky Normal Form]

Convert the following context-free grammar $G = (\{S, A\}, \{a, b, c\}, P, S)$ into an equivalent grammar that is in Chomsky Normal Form using the construction detailed in the proof of Theorem 3.9, where P_1 is defined as follows:

$$P = \{S \to cS, S \to aAb, S \to ab, A \to aAb, A \to ab, A \to cc\}.$$