Automata and Grammars

SS 2018

Assignment 7

Solutions are to be presented at the **Seminary** on **Thursday, April 12, 2018**.

Problem 7.1 [Pumping Lemma]

Prove that the following languages are not regular by applying the Pumping Lemma for regular languages (Theorem 2.34):

(a) $L_a = \{ a^m b^n \mid n > m \geq 1 \}$,
(b) $L_b = \{ a^m b^n \mid m \leq n \leq 2m \}$,
(c) $L_c = \{ w \in \{a,b\}^* \mid |w|_a = |w|_b \}$,
(d) $L_d = \{ a^{2^n} \mid n \geq 0 \}$.

Problem 7.2. [Pumping Lemma]

Let L be the following language over $\Sigma = \{a, b\}$:

$$L = \{ ab(ba)^m ba^n ba^n \mid m, n \geq 1 \} \cup \{ abba^m ba^n \mid m, n \geq 1 \}.$$

(a) Prove that L satisfies the Pumping Lemma for regular languages (Theorem 2.34) by determining a corresponding constant $c \geq 1$.

(b) Prove that the language L is not regular by using the Theorem of Myhill-Nerode (Theorem 2.12).

(c) Is the language $L_c = \{ (ab)^n a(ba)^n \mid n \geq 1 \}$ regular? Provide a proof for your answer!

Problem 7.3. [Decision Problems]

Determine the cardinality of the language $L(A_i)$ for the following NFAs A_i $(1 \leq i \leq 4)$:

(a) $A_1 = (Q, \{a,b\}, \delta_1, \{0,5\}, \{6\})$, where δ_1 is defined as follows:

δ_1	$0^{\leftarrow}$	1	2	3	4	$5^{\leftarrow}$	$6_{\rightarrow}$	7	8
a	4	2	3	1	0	7	3	1	3
b	1	3	2	3	4	8	–	8	6

(b) $A_2 = (Q, \{a,b\}, \delta_2, \{0\}, \{5,10\})$, where δ_2 is defined as follows:

δ_2	$0^{\leftarrow}$	1	2	3	4	$5_{\rightarrow}$	6	7	8	9	$10_{\rightarrow}$
a	1	3	1	4	2	4	2	6	8	6	10
b	7	1	–	3	4	10	0	9	7	–	5

(c) $A_3 = (Q, \{a,b\}, \delta_3, \{0\}, \{0,5,7\})$, where δ_3 is defined as follows:

δ_3	$0^{\leftarrow}_{\rightarrow}$	1	2	3	4	$5_{\rightarrow}$	6	$7_{\rightarrow}$	8	9
a	1,2	–	–	2	6	4	2,7	–	5,9	9
b	4	1,3	2	–	–	3	–	8,9	–	9

(d) $A_4 = (Q, \{a,b\}, \delta_4, \{0,5\}, \{10\})$, where δ_4 is defined as follows:

δ_4	$0^{\leftarrow}$	1	2	3	4	$5^{\leftarrow}$	6	7	8	9	$10_{\rightarrow}$
a	1	2	3	4	3	–	–	–	3,9	–	–
b	–	–	–	9	–	6	7	8	–	1,6,10	–