6

The Einstein–Podolsky–Rosen Paradox

My interpretation of the repeated measurement experiments in section 4.2 was:

> An atom with a definite value of m_z doesn't have a definite value of m_x. All that can be said is that when m_x is measured, there is probability $\frac{1}{2}$ of finding $+m_B$ and probability $\frac{1}{2}$ of finding $-m_B$.

This is in many ways the simplest and most natural interpretation, but there are other possibilities. For example, the "measurement disturbs a classical system" possibility:

> An atom with a definite value of m_z also has a definite value of m_x, but the measurement of m_z disturbs the value of m_x in an unpredictable way.

or the "complex atom" possibility:

> An atom with a definite value of m_z also has a definite value of m_x, but this value changes so rapidly that no one can figure out what that value is.

The Einstein–Podolsky–Rosen (or EPR) argument shows that both of these "other interpretations" are untenable.

I will give the argument in the form of two hypothetical experiments. Because of technical difficulties, these experiments have never been carried out in exactly the form that I will describe. But similar experiments have been performed, most notably by Alain Aspect and his collaborators at the University of Paris's Institute of Theoretical and Applied Optics at Orsay. Figure 6.1 shows the apparatus that this group employed and, as usual, it is much more elaborate than the sketch diagrams that I will use later to describe the hypothetical experiment. Our hypothetical experiment

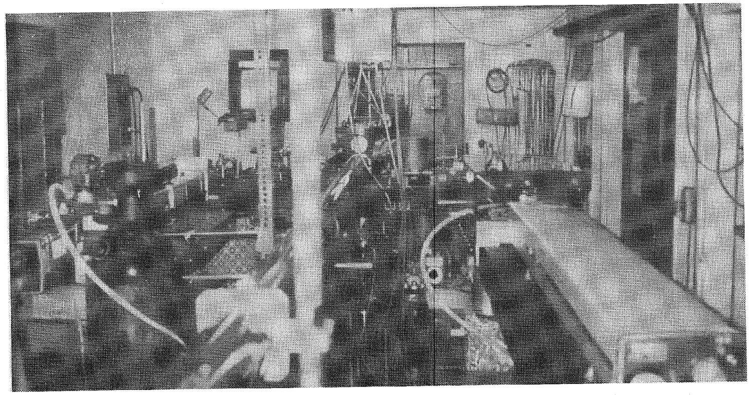


Fig. 6.1. Alain Aspect's laboratory in Orsay, France (courtesy of A. Aspect).

will employ a pair of atoms and detectors that tilt by 120°. Aspect's real experiment employed a pair of "photons" ("particles of light") and detectors that tilted by 22.5°. In spite of these technical differences, the real experiment was *conceptually* equivalent to the one I will describe here, and its results are a ringing endorsement of quantum mechanics.

Locality

Before proceeding, I must attend to one small but essential point: the term "local". It is clear that something which happens at one place can influence what happens far away. For example, a newspaper article printed in Madrid can foment a revolution in Buenos Aires. But the effect happens some time after the cause, because it takes some time for the agent of influence (the newspapers) to travel from Madrid to Buenos Aires, and as they travel they always move bit by bit — they never disappear from one place and reappear at another without passing through intermediate points. This method of influence is called "local". Modern communication technology might appear to be non-local, because when you speak into a telephone it seems that you can be heard far away at the same instant. But in fact there is a short — and usually unnoticeable — delay between the speaking and the hearing, as electrical signals encoding your voice travel through telephone lines at the speed of light.

> *Technical aside:* Notice that the very definition of locality involves concepts like cause and effect, concepts that assume a deterministic world. Because quantum mechanics is not deterministic and events can take place without causes, the concept

of locality becomes more subtle and complex. The technical literature is thus full of terms like "active locality", "passive locality", "non-locality", and "alocality".

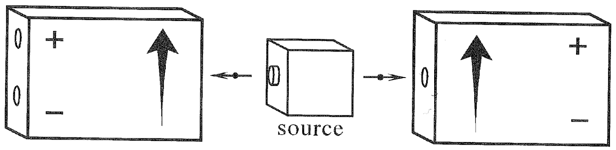
The assumption of locality is so natural and commonplace that it has been enshrined in poetry:[*]

> And when the loss has been disclosed, the Secret Service say:
> "It *must* have been Macavity!" — but he's a mile away.

Einstein's theory of relativity puts the assumption of locality on an even firmer basis, establishing that no causal agent can travel faster than a light signal. Standard quantum mechanics, as presented in this book, retains the assumption of locality. But it *is* possible to produce alternatives to standard quantum theory that are non-local.

I mention locality here because the experiments described below illuminate our old ideas in a strange — but ultimately satisfying — new light.

6.1 Experiment 6.1: Distant measurements



In this experiment a box labeled "source" produces a pair of atoms with a net magnetic arrow of zero, and the two atoms fly off in opposite directions. Each atom is detected by its own vertical Stern–Gerlach analyzer.

Observed results: The probability that the right atom leaves through the $+$ exit is $\frac{1}{2}$, the probability that it leaves through the $-$ exit is $\frac{1}{2}$. Similarly for the left atom. But if the right atom leaves through its $+$ exit, then the left atom always leaves through its $-$ exit, and vice versa. This is true regardless of which, if either, analyzer is closer to the source. It is also true regardless of the orientation of the two analyzers, as long as both have the same orientation.

Here is a straightforward proposal that explains most of these observations: Simply suppose that when the pairs of atoms are produced, one atom has $m_z = +m_B$ and the other has $m_z = -m_B$. This proposal explains the first four observations but it is inconsistent with the last one. If the two analyzers are, say, horizontal instead of vertical, then under this proposal

[*] T.S. Eliot, *Old Possum's Book of Practical Cats.*

it would be possible (see problem 6.2) for both atoms to leave through their $+$ exits, or for both to leave through their $-$ exits. But in fact the two atoms always leave through opposite exits. The straightforward proposal is appealing, but it must be wrong. Eventually we will replace the straightforward yet incorrect proposal with a much more elaborate one, a proposal called "quantum mechanics". For the time being, however, it is important to get a clear idea of how atoms *actually do* behave before rushing into new proposals. So how do atoms behave?

Imagine, for example, that the left analyzer is five miles from the source, while the right analyzer is five miles plus one inch from the source. Then the left atom will go into its analyzer and be measured before the right atom goes into its analyzer. Suppose that the left atom leaves the $+$ exit. Then it is known with certainty that the right atom has $m_z = -m_B$ (i.e. that when it gets to its analyzer it will leave through the $-$ exit), but the right atom itself has not been measured. It is impossible that the right atom, ten miles away from the scene of the measurement, could have been mechanically disturbed by the measurement of the left atom.[†] The first alternative interpretation mentioned on page 38 must be rejected.

If you are familiar with Einstein's theory of relativity, you know that the fastest possible speed at which a message can travel is the speed of light. Yet this experiment suggests a mechanism for instantaneous communication: When the two atoms are launched, it cannot be predicted whether the right atom will leave the $+$ exit or the $-$ exit once it gets to its analyzer. But the instant that the left atom leaves the $+$ exit of its analyzer, it is known that the right atom (now ten miles away) will leave the $-$ exit once it gets to the right analyzer. This seems to be instantaneous communication. But the important point is not whether "it is known that the right atom will leave the $-$ exit" but rather *who* knows that the right atom will leave the $-$ exit. Certainly the person standing next to the left-hand analyzer knows it[‡] but the person on the left won't be able to tell the person on the right except through some ordinary, slower-than-light mechanism. The result is strange (Einstein called it "spooky") but it does not open up the door to instantaneous communication.

Quantum mechanics forces us to the brink of implausibility — *but not beyond.*

> *Technical aside:* The conceptual equivalent of this experiment has been performed many times, usually with detectors located yards rather than miles apart. But in 1997 Nicolas Gisin of

[†] Is it really "impossible"? In fact, this is the assumption of locality which, as I have mentioned, is very natural but nevertheless an assumption.

[‡] And the person standing next to the right-hand analyzer knows that the person standing next to the left-hand analyzer knows it.

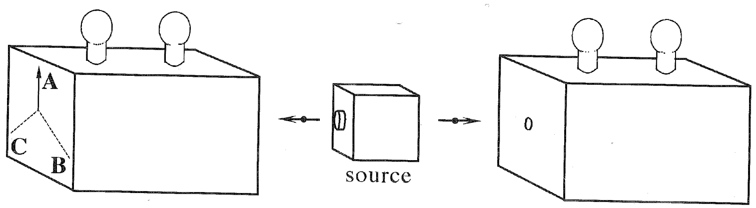
the University of Gèneva and his collaborators performed the experiment with detectors in the Swiss villages of Bellevue and Bernex, separated by nearly seven miles.

6.2 Experiment 6.2: Random distant measurements

This experiment is called the "test of Bell's theorem". The reasoning is intricate, so I give an outline here before plunging into the details. We will build an apparatus much like the previous one with a central source that produces a pair of atoms, and with two detector boxes. Mounted atop each detector box are a red lamp and a green lamp. Every time the experiment is run, a single lamp on each detector box lights up. On some runs the detector on the left flashes red and the detector on the right flashes green, on other runs both detectors flash red, etc. When the apparatus is analyzed by quantum mechanics, we find that the probability of each detector flashing a different color is $\frac{1}{2}$. But we can also analyze the apparatus under the assumption of local determinism. This analysis shows that the probability of each detector flashing a different color is $\frac{5}{9}$ or more. (Exactly how much more depends on exactly which local deterministic scheme is employed, see problem 6.4.) Experiment agrees with quantum mechanics, so the assumption of local determinism, natural though it may be, is false. Any local deterministic scheme, including the second alternative interpretation mentioned on page 38, must be wrong.

The apparatus

This experiment uses the same source as the previous experiment, but now the detectors are not regular Stern–Gerlach analyzers, but the tilting Stern–Gerlach analyzers described in section 5.3 (page 33). Each of the two analyzers has probability $\frac{1}{3}$ of being oriented as **A**, **B**, or **C**. If you wish, you may set the detector orientations and then have the source generate its pair of atoms, but you will get the same results if you first launch the two atoms and then set the detector orientations while the atoms are in flight. Mounted on each detector are two colored lamps. If an atom comes out of the + exit, the red lamp flashes; if an atom comes out the − exit, the green lamp flashes.



The prediction of quantum mechanics

If the two detectors happen to have the same orientation, then this experiment is exactly the same as the previous one, so exactly the same results are obtained: the two detectors always flash different colors. On the other hand, if the two detectors have different orientations, then they might or might not flash different colors.

What is the probability that the two detectors flash different colors in general, that is, when the two detectors might or might not have the same orientation? Suppose the detector on the left is closer to the source than the detector on the right. If the left detector were set to **A** and flashed green (that is, −), then the atom on the right has $m_z = +m_B$. In the previous chapter we saw that when such an atom enters the right detector, it has probability $\frac{1}{2}$ of causing a red flash and probability $\frac{1}{2}$ of causing a green flash. You can readily generalize this reasoning to show that regardless of orientation, the two detectors flash different colors with probability $\frac{1}{2}$.

We conclude that:

(1) If the orientation settings are the same, then the two detectors flash different colors always.
(2) If the orientation settings are ignored, then the two detectors flash different colors with probability $\frac{1}{2}$.

And these results are indeed observed!

The prediction of local determinism

In any local deterministic scheme, each atom must leave the source already supplied with an instruction set that determines which lamp flashes for each of the three orientation settings. For example, an instruction set might read (if set to **A** then flash red, if set to **B** then flash red, if set to **C** then flash green), which we abbreviate as (RRG). One natural way to implement an instruction set scheme would be through the atom's associated magnetic arrow: if the detector is vertical (orientation **A**) and the atom's arrow points anywhere north of the equator, then the atom leaves through the + exit, while if the atom's arrow points anywhere south of the equator, then the atom leaves through the − exit. Similar rules hold for orientations **B** and **C**: the atom always leaves through the exit towards which its arrow most closely points.§ The argument that follows holds for

§ This postulated scheme is inconsistent with quantum mechanics because it assumes that an atom's magnetic arrow points in the same manner that a classical stick does, with definite values for all three projections m_x, m_y, and m_z simultaneously.

this natural scheme, but it also holds for any other oddball instruction set scheme as well.

To explain observation (1) above, assume that the two atoms are launched with opposite instruction sets: if the atom going left is (GRG), then the atom going right is (RGR), and so forth. (In the "natural" scheme, the two atoms are launched with magnetic arrows pointing in opposite directions.) Now let's see how we can explain observation (2).

If the instruction set for the atom going left is (RRG), and for the atom going right is (GGR), then what colors will the detectors flash? That depends on the orientation settings of the two detectors. Suppose the left detector were set to **C** and the right detector were set to **A**. Then the third letter of (RRG) tells us that the left detector would flash green, and the first letter of (GGR) tells us that the right detector would flash green. The same list-lookup reasoning can be applied to any possible orientation setting to produce the following table.

orientation settings	detectors flash
AA	RG: different
BB	RG: different
CC	GR: different
AB	RG: different
BA	RG: different
BC	RR: same
CB	GG: same
AC	RR: same
CA	GG: same

There are nine possible orientation settings and five of them lead to different color flashes. So if the atom going left is (RRG), then the probability of different color flashes is $\frac{5}{9}$. A little thought shows that the same result applies if the atom going left is (GGR), or (GRG), or anything but (RRR) and (GGG). In the last two cases, the probability of different color flashes is of course 1.

Now we know the probability of different color flashes for any given instruction set. We want to find the probability of different color flashes period. To calculate this we need to know what kind of atoms the source makes. (If it makes only (RRR)s paired with (GGG)s then the probability of different color flashes is 1. If it makes only (RRG)s paired with (GGR)s then the probability of different color flashes is $\frac{5}{9}$. If it makes [(RRR) paired with (GGG)] half the time and [(RRG) paired with (GGR)] half the time, then the probability of different

color flashes is half-way between $\frac{5}{9}$ and 1.) Because I don't know exactly how the source works, I can't say exactly what the probability for different color flashes is. But I do know that any source can make only eight kinds of atoms, because only eight kinds of atoms exist:

kind of atom going left	probability of different color flashes
(RRR)	1
(GGG)	1
(RRG)	5/9
(RGR)	5/9
(GRR)	5/9
(RGG)	5/9
(GRG)	5/9
(GGR)	5/9

Thus for any kind of source, the probability of different color flashes is some mixture of probability 1 and probability $\frac{5}{9}$.

We conclude that in any instruction set scheme, the detectors will flash different colors with probability $\frac{5}{9}$ (55.5%) *or more.*

The conclusion

But in fact, the detectors flash different colors with probability $\frac{1}{2}$! The assumption of local determinism has produced a conclusion which is violated in the real world, and hence it must be wrong. Probability is not just the *easiest* way out of the conundrum of projections, it is the *only* way out.

> *Technical aside:* What, only? Well, almost only. In fact, our arguments only rule out the existence of instruction sets, and hence it permits alternatives to standard probabilistic quantum theory that do not rely on instruction sets. David Bohm, and others, have invented such deterministic but non-local alternatives. If you dislike quantum mechanics because it's too weird for your tastes, this may make you happy. However, these alternative theories are necessarily pretty weird themselves. For example, in Bohm's theory the two atoms don't need instruction sets because they can communicate with each other instantaneously. To be absolutely accurate, probability is the only *local* way out of the conundrum of projections.

6.3 References

The subject of this chapter has a rich intellectual heritage. The general idea was introduced in

A. Einstein, B. Podolsky, and N. Rosen, "Can quantum-mechanical description of physical reality be considered complete?", *Physical Review*, **47** (1935) 777–780,

and the specific form of our experiment 6.1 was devised by

David Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, New Jersey, 1951) pages 611–623.

But the most powerful part of the argument, the one embodied in our experiment 6.2, was developed by John Bell in 1964 and is called "Bell's theorem". Bell's writings on quantum mechanics, ranging from the popular to the very technical, are collected in

John S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, UK, 1987).

The best semi-popular treatment of Bell's theorem and the Einstein–Podolsky–Rosen paradox (and the inspiration for much of this chapter) is

N.D. Mermin "Is the moon there when nobody looks? Reality and the quantum theory", *Physics Today*, **38** (4) (April 1985) 38–47; see also the letters reacting to this article: *Physics Today*, **38** (11) (November 1985) 9–15, 136–142.

A computer program to simulate this test of Bell's theorem is

Darrel J. Conway, *BellBox* (Physics Academic Software, Raleigh, North Carolina, 1993).

Real experimental results mentioned in this chapter were reported in

Alain Aspect, Philippe Grangier, and Gérard Roger, "Experimental realization of Einstein–Podolsky–Rosen–Bohm *gedankenexperiment*: A new violation of Bell's inequalities", *Physical Review Letters*, **49** (1982) 91–94,
Alain Aspect, Jean Dalibard, and Gérard Roger, "Experimental test of Bell's inequalities using time-varying analyzers", *Physical Review Letters*, **49** (1982) 1804–1807,
W. Tittel, J. Brendel, B. Gisin, T. Herzog, H. Zbinden, and N. Gisin, "Experimental demonstration of quantum correlations over more than 10 km", *Physical Review* A, **57** (1998) 3229–3232,

but these papers are difficult for non-physicists to read. You might want to look instead at the reviews

Arthur L. Robinson, "Quantum mechanics passes another test", *Science*, **217** (30 July 1982) 435–436,
Arthur L. Robinson, "Loophole closed in quantum mechanics test", *Science*, **219** (7 January 1983) 40–41,
Andrew Watson, "Quantum spookiness wins, Einstein loses in photon test", *Science*, **277** (25 July 1997) 481.

A recent high-accuracy test of Bell's theorem is described in

P.G. Kwiat, E. Waks, A.G. White, I. Appelbaum, and P.H. Eberhard, "Ultra-bright source of polarization-entangled photons", *Physical Review* A, **60** (August 1999) R773–R776.

6.4 Problems

6.1 *Instantaneous communication.* In your own words, explain why you cannot send a message instantaneously using the mechanism of experiment 6.1. If quantum mechanics were deterministic rather than probabilistic, yet the distant atoms still always left from opposite exits, would you then be able to send a message instantaneously? What if the operator of the left-hand Stern–Gerlach analyzer were somehow¶ able to force his atom to come out of the + exit? (You might want to answer by completing the following story: "An eccentric gentleman in London has two correspondents: Ivan in Seattle and Veronica in Johannesburg. Every Monday he sends each correspondent a letter, and the two letters are identical except that he signs one in red ink and one in green ink. The instant that Veronica opens her letter, she knows ....")

6.2 *Quantal states for distant measurements.* Mr. Parker is an intelligent layman. He is interested in quantum mechanics and is open to new ideas, but he wants evidence before he will accept wild-eyed assertions. "I like the argument of experiment 6.1," he says, "but I don't like the idea that when the left atom is detected, the right atom instantly jumps into the state with $m_z = -m_B$. I think that one atom is produced in the state $m_x = +m_B$ and the other atom is produced in the state $m_x = -m_B$, and that there are no instant state jumps." Show that Mr. Parker's suggestion is consistent with the

¶ Perhaps by magic powers, but not so magic as to change the fact that the two atoms always leave from opposite exits.

observation that "the right atom leaves the + exit with probability $\frac{1}{2}$, and similarly for the left atom". However, show also that if it were true, then on about $\frac{1}{4}$ of the experimental runs, both atoms would emerge from their respective + exits.

6.3 *A probability found through quantum mechanics.* In the test of Bell's theorem, experiment 6.2, what is the probability given by quantum mechanics that, if the orientation settings are different, the two detectors will flash different colors?

6.4 *A probability found through local determinism.* The experimental test of Bell's theorem shows that the postulated instruction sets do not exist. But suppose that they did. Suppose further that a given source produces the various possible instruction sets with the probabilities listed below:

kind of atom going left	probability of making such a pair
(RRR)	1/2
(RRG)	1/4
(GRR)	1/8
(RGG)	1/8

If this particular source were used in experiment 6.2, what would be the probability that the detectors flash different colors? Hint: Compare the draft lottery problem 5.6.

7

Variations on a Theme by Einstein

The previous chapter covered the most important aspects of the Einstein–Podolsky–Rosen conundrum. But some interesting new features have come up since Aspect performed his experiments, and I thought you might enjoy them, so I'll mention two of them here. You may skip this chapter without interrupting the flow of the book's argument.

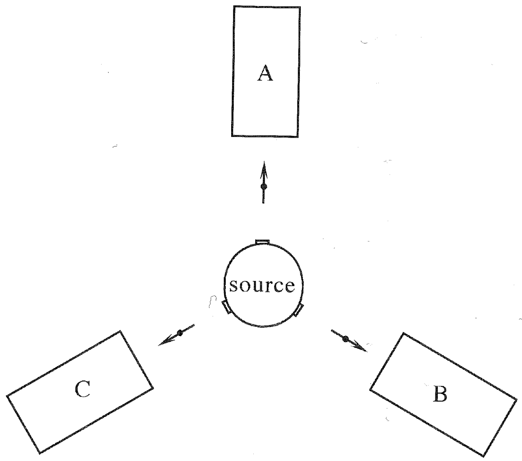
The results of the Aspect experiment were welcomed by most scientists as a final confirmation of the principles of quantum mechanics, principles that had already been verified magnificently in numerous experiments that were not as clean nor as easy to understand as the test of Bell's theorem. But scientists also looked for possible flaws in the confirmation, and they found one. We have discussed an ideal experiment, in which the source produces a pair of atoms and each tilting analyzer detects one of them. But in Aspect's real experiment, it often happened that after the source launched its atoms only one of the two atoms was detected, and sometimes neither of them were. This is not surprising: perhaps one of the atoms collided with a stray nitrogen molecule and was deflected away from its detector, or perhaps the detector electronics were pausing to reset after detecting one atom when a second atom rushed in. For these reasons, in analyzing his experiment Aspect ignored cases where only one atom was detected. But another possibility is that each atom is generated with an instruction set which could include the instruction "don't detect me". If this possibility is admitted, then one can invent local deterministic schemes that are consistent with Aspect's experimental results.

Personally, I regard this objection as far-fetched. But either of the two proposed experiments described here would overrule this objection definitively, because both of them produce situations in which quantum mechanics predicts that something might happen, whereas local determinism predicts that the same thing will *never* happen. Neither experiment has been executed in its entirety, but work is in progress on both and the preliminary results annoûnced to date support quantum mechanics and oppose local determinism.

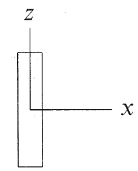
7.1 The Greenberger–Horne–Zeilinger variation on the Einstein–Podolsky–Rosen experiment

This experiment involves a source that ejects three atoms in an initial state that is hard to produce and even harder to describe. It is impossible for me to justify the prediction of quantum mechanics in a book at this level. For these two reasons I considered ignoring this experiment altogether in writing this book. But there is a payoff so rich that I had to include it: Whereas the test of Bell's theorem gives a circumstance in which the quantal probability for something happening is 50% while the local deterministic probability is more than 55%, the Greenberger–Horne–Zeilinger (or GHZ) variation gives a circumstance in which the quantal probability is 1 and the local deterministic probability is 0.

A top view of the Greenberger–Horne–Zeilinger experiment is sketched below. The source ejects three atoms in a special state, and each atom flies



off to its own detector. Like the detectors in the test of Bell's theorem, each box contains a Stern–Gerlach analyzer that can be tilted and set to various orientations. But unlike the tilting analyzers used before, these analyzers can be set to only two orientations: the z direction (vertical) or the x direction (horizontal).



Back panel of each Greenberger–Horne–Zeilinger detector, showing the two orientations for its internal Stern–Gerlach analyzer. This analyzer is set to orientation z.

The orientations of the three analyzers are reported through a code like xxz, which means that detectors A and B are set to x while detector C is set to z. As with the test of Bell's theorem (experiment 6.2, page 42), the detector orientations can be set after the atoms have been launched, while they are still in flight toward the detectors.

The predictions of quantum mechanics are:

	detector settings	what happens
(1)	zxx	odd number (1 or 3) go to +
(2)	xxz	odd number (1 or 3) go to +
(3)	xzx	odd number (1 or 3) go to +
(4)	zzz	even number (0 or 2) go to +
(5)	other	not used in this argument

Thus whenever two analyzers are set to x and one to z, either all three atoms leave through the + exits of their respective analyzers, or else one leaves through the + exit and the other two leave through − exits.

The argument for instruction sets

I will give an argument based on line (1) of the prediction that makes it seem reasonable that each atom is launched from the source with an instruction set, so that it will know whether to go to + or to − when it reaches its detector, regardless of what the settings of the detectors are. If you find this assertion reasonable already, you may skip the argument. Remember, however, that quantum mechanics maintains that this natural surmise is *not* correct, because an atom with a definite value of m_x does not have a definite value of m_z.

Suppose that I wished to measure the value of m_x for the atom going to detector C. One way to do it would be by setting A to z, B to x, and C to x, corresponding to line (1) of the prediction. Then I ask what would happen if I used only the detectors at A and at B, and forgot about the detector at C. (This despite the fact that it is the atom going to C that I'm interested in.) If detector A (set to z) measured +, and detector B (set to x) measured $+$, then detector C (set to x) would have to measure $+$ as well, because according to line (1) of the prediction there must be either one or three atoms going to $+$. So if the atoms going to A and B come out through the $+$ exit, then I don't need to actually measure m_x of the atom going to C — I know what's going to happen at C merely by observing what had happened at A and B.

In fact, the same is true regardless of how the atoms come out at A and B, as long as the detectors are set to zxx:

	outcomes at A	outcomes at B		outcome at C
given	$+$	$+$	then	$+$
	$+$	$-$		$-$
	$-$	$+$		$-$
	$-$	$-$		$+$

In short, if the settings are zxx, then by reading the outcomes at A and B, I can determine the outcome at C. I don't need to actually put an analyzer at C. The same is true for other directions: reading the outcomes at B and C enables me to determine the outcome at A, and reading the outcomes at A and C enables me to determine the outcome at B. And a glance at the quantal prediction on page 51 will convince you that parallel statements hold if the settings are xxz or xzx. In short, lines (1), (2), and (3) of the prediction enable you to determine either m_z or m_x of any atom merely by measuring appropriate quantities for the other two atoms, without actually touching the atom in question.

Because the detectors don't communicate with each other, the natural interpretation of this fact is that when an atom is launched from the source, it must already "know" how it will behave at the detector, regardless of the setting of that detector. Such an "instruction set" might be encoded into the direction of the atom's magnetic arrow, but it could conceivably be encoded in some strange or complicated way. In what follows I make no assumption about how the instruction set is encoded, only that it exists.

The prediction of local determinism

I will write down the instruction set of all three atoms using a symbol like

$$\begin{array}{c} \text{atom heading toward} \\ \begin{array}{ccc} \mathrm{A} & \mathrm{B} & \mathrm{C} \end{array} \\ \begin{pmatrix} + & - & - \\ - & - & + \end{pmatrix} \end{array} \quad \begin{array}{l} \leftarrow \text{if set to } z \\ \leftarrow \text{if set to } x \end{array}$$

This notation means that the atom heading toward detector A will leave through the $+$ exit if that detector is set to z, through the $-$ exit if it is set

to x. The atom heading toward detector B will leave the $-$ exit regardless of setting. The atom heading toward detector C will leave the $-$ exit if that detector is set to z, the $+$ exit if it is set to x.

Now I ask: What instruction sets are consistent with the quantal prediction? We will examine the first four lines of the table on page 51 in turn.

Line (1) of the table pertains to detector settings zxx, so it has nothing to say about what will happen if A is set to x, if B is set to z, or if C is set to z. In the following table the instructions for such settings are set to "?". Notice from the table that with these settings either one or three atoms leave through the $+$ exit, and therefore the only instruction sets compatible with line (1) are the following:

instruction sets consistent with line (1)

$$\begin{pmatrix} + & ? & ? \\ ? & - & - \end{pmatrix} \quad \begin{pmatrix} - & ? & ? \\ ? & + & - \end{pmatrix}$$

$$\begin{pmatrix} - & ? & ? \\ ? & - & + \end{pmatrix} \quad \begin{pmatrix} + & ? & ? \\ ? & + & + \end{pmatrix}$$

Which of these instruction sets is consistent with line (2) of the quantal prediction as well? We begin by considering only instruction sets of the type shown in the upper left above. Line (2) involves the setting xxz, so this reasoning will enable us to fill in the x (bottom) slot of column A and the z (top) slot of column C. We already know, from the entry above, that the atom heading for detector B will come out through the $-$ exit. Since a total of either one or three atoms must come out through the $+$ exit in this circumstance, then of the atoms heading for A and C, one must come out through $+$ and the other through $-$. Thus the instruction set must be either

$$\begin{pmatrix} + & ? & + \\ - & - & - \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} + & ? & - \\ + & - & - \end{pmatrix}.$$

The same game can be played with the other three types of instruction sets consistent with line (1), resulting in:

instruction sets consistent with lines (1) and (2)

$$\begin{pmatrix} + & ? & + \\ - & - & - \end{pmatrix} \quad \begin{pmatrix} + & ? & - \\ + & - & - \end{pmatrix} \quad \begin{pmatrix} - & ? & - \\ - & + & - \end{pmatrix} \quad \begin{pmatrix} - & ? & + \\ + & + & - \end{pmatrix}$$

$$\begin{pmatrix} - & ? & - \\ + & - & + \end{pmatrix} \quad \begin{pmatrix} - & ? & + \\ - & - & + \end{pmatrix} \quad \begin{pmatrix} + & ? & - \\ - & + & + \end{pmatrix} \quad \begin{pmatrix} + & ? & + \\ + & + & + \end{pmatrix}$$

From here it is easy to find the instruction sets consistent with line (3) of the quantal prediction as well:

instruction sets consistent with lines (1), (2), and (3)

$$\begin{pmatrix} + & + & + \\ - & - & - \end{pmatrix} \quad \begin{pmatrix} + & - & - \\ + & - & - \end{pmatrix} \quad \begin{pmatrix} - & + & - \\ - & + & - \end{pmatrix} \quad \begin{pmatrix} - & - & + \\ + & + & - \end{pmatrix}$$

$$\begin{pmatrix} - & + & - \\ + & - & + \end{pmatrix} \quad \begin{pmatrix} - & - & + \\ - & - & + \end{pmatrix} \quad \begin{pmatrix} + & - & - \\ - & + & + \end{pmatrix} \quad \begin{pmatrix} + & + & + \\ + & + & + \end{pmatrix}$$

These eight, now completely determined, instruction sets are the only ones consistent with the quantal predictions given in lines (1), (2), and (3).

Which of these eight instruction sets is consistent with line (4) as well? In line (4) the detectors are set to zzz, so only the upper row of the instruction sets are relevant. The instruction set shown in the upper left above would result in all three atoms leaving through the + exits of their analyzers. But according to quantum mechanics (see line (4) of the quantal prediction on page 51), in this case an even number of atoms must leave + exits. Three is an odd number, so the instruction set in the upper left above must be ruled out as inconsistent with the predictions of quantum mechanics. The instruction set in the lower right must be ruled out for the same reason. All the remaining instruction sets call for exactly one of the three atoms to leave through + exits. But one is also an odd number! In short:

instruction sets consistent with lines (1), (2), (3), and (4)

NONE!

Once again, the existence of instructions sets — regardless of how subtly the instructions are encoded — is inconsistent with the predictions of quantum mechanics.

7.2 Hardy's variation on the Einstein–Podolsky–Rosen experiment

This variation is harder to describe and I will not treat it in detail. It involves a source that ejects two atoms toward two different detectors, each of which can be tilted to two different angles, and an unusual initial state at the source. The experiment looks for a certain combination of events. The local deterministic prediction is that this combination will never happen. The quantal prediction is that it will happen with a

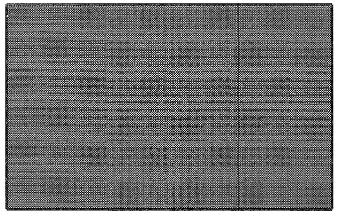


Fig. 7.1. The golden rectangle.

probability of 9.017%. Thus if the combination happens in an experiment even once, then local determinism must be wrong.

One thing that intrigues me about this variation is the mathematical origin of the probability 0.09017.... The number is g^5, where the constant g is equal to $(\sqrt{5}-1)/2 = 0.6180\ldots$ and is called "the golden mean". If a line of length 1 is divided into two pieces so that the ratio of the length of the whole to the length of the long piece is equal to the ratio of the length of the long piece to the length of the short piece, then the long piece will have length g. The ancient Greeks considered a rectangle of width 1 and height g to be the "ideal" (most beautiful) rectangle. The Parthenon in Athens, for example, has a height of g times its width. Rectangles with these proportions also appear in the work of Leonardo da Vinci, Titian, and Mondrian. In addition the number is connected with the Great Pyramid, the star pentagram (which in one form appears in the American flag and which in another is said to call up the devil), the Fibonacci sequence, recursion relations, and with algorithms for locating the minimum of a one-variable function. But this is the first time I've ever seen it appear in quantum mechanics.

7.3 References

For the Greenberger–Horne–Zeilinger variation, see

Daniel M. Greenberger, Michael A. Horne, Abner Shimony, and Anton Zeilinger, "Bell's theorem without inequalities", *American Journal of Physics*, **58** (1990) 1131–1143,

N.D. Mermin, "The (non)world (non)view of quantum mechanics", *New Literary History*, **23** (1992) 855–875,

D. Bouwmeester, J.-W. Pan, M. Daniell, H. Weinfurter, and A. Zeilinger, "Observation of three-photon Greenberger–Horne–Zeilinger entanglement", *Physical Review Letters*, **82** (1999) 1345–1349.

For Hardy's variation, see

Lucien Hardy, "Nonlocality for two particles without inequalities for almost all entangled states", *Physical Review Letters*, **71** (1993) 1665–1668,

David Branning, "Does nature violate local realism?", *American Scientist*, **85** (1997) 160–167.

A technical but insightful exchange concerning Hardy's variation and its implications for locality in quantum mechanics is

Henry P. Stapp, "Nonlocal character of quantum theory", *American Journal of Physics*, **65** (1997) 300–304,

N. David Mermin, "Nonlocal character of quantum theory?", *American Journal of Physics*, **66** (1998) 920–924,

Henry P. Stapp, "Meaning of counterfactual statements in quantum physics", *American Journal of Physics*, **66** (1998) 924–926.

8
Optical Interference

8.1 Overview

We have uncovered the first central principle of quantum mechanics, which is that *the outcome of an experiment cannot, in general, be predicted exactly; only the probabilities of the various outcomes can be found.* In particular, for the magnetic arrow of a silver atom, we know:

> If m_z has a definite value, then m_x doesn't have a value. If you measure m_x, then of course you find some value, but no one (not even the atom itself!) can say with certainty what that value will be — only the probabilities of measuring the various values can be calculated.

How do you like it? Do you feel liberated from the shackles of classical determinism? Or do you feel like Matthew Arnold, who wrote in *Dover Beach* that

> ... the world, which seems
> To lie before us like a land of dreams,
> So various, so beautiful, so new,
> Hath really neither joy, nor love, nor light,
> Nor certitude, nor peace, nor help from pain;
> And we are here as on a darkling plain
> Swept with confused alarms of struggle and flight,
> Where ignorant armies clash by night.

Regardless of your personal reaction, it is our job as scientists to *describe* nature, not to dictate to it!

In particular, we know that the model of a magnetic needle as an arrow, so carefully developed in chapter 2 and so correct within the domain of classical mechanics, must be wrong. In classical mechanics, magnetic