Automata and Grammars

SS 2018

Assignment 6

Solutions are to be presented at the **Seminary** on **Thursday, April 5, 2018**.

Problem 6.1 [Right and Left Quotients]

Let L and P be two languages over Σ. The *right quotient* L/P of L by P is defined as

$$L/P = \{\, u \in \Sigma^* \mid \exists\, v \in P : uv \in L \,\}$$

and the *left quotient* $P\backslash L$ of L by P is defined as

$$P\backslash L = \{\, v \in \Sigma^* \mid \exists\, u \in P : uv \in L \,\}.$$

Let $A = (Q, \Sigma, \delta, I, F)$ be an NFA such that $L(A) = L$.

(a) Prove that the right quotient L/P is a regular language by showing how to obtain an NFA for L/P from A and P.

(b) Prove that the left quotient $P\backslash L$ is a regular language by showing how to obtain an NFA for $P\backslash L$ from A and P.

Problem 6.2. [Regular Expressions]

(a) Write a regular expression that defines the language on $\{a, b\}$ that contains exactly those words that begin with 'ba' and that end with 'ab'.

(b) Write a regular expression that defines the language on $\{a, *\}$ that contains exactly all those words of the form a, $a * a$, $a * a * a$,...

(c) Convert the following regular expressions into equivalent ε-NFAs by using the construction from the proof of Theorem 2.30:

- $(ab + c)^*$,
- $((ab + c)^* a (bc)^* + b)^*$.

Problem 6.3. [Regular Expressions]

Convert the NFA $A = (\{q_0, q_1, q_2\}, \{a, b\}, q_0, \{q_1, q_2\}, \delta)$ into an equivalent regular expression using the construction from the proof of Theorem 2.31, where δ is defined as follows:

δ	$q_0^{\leftarrow}$	$q_{1\rightarrow}$	$q_{2\rightarrow}$
a	q_0	q_2	q_0
b	q_1	q_1	q_1

Problem 6.4. [Regular Expressions]

Prove that the following identities hold for all regular expressions r, s, t:

$$
\begin{array}{lll}
(1) & r + \emptyset & = & r, \\
(2) & r\varepsilon & = & r, \\
(3) & r\emptyset & = & \emptyset, \\
(4) & r(s+t) & = & (rs) + (rt), \\
(5) & \emptyset^* & = & \varepsilon, \\
(6) & (r^*)^* & = & r^*, \\
(7) & (r+s)^* & = & (r^* + s^*)^*,
\end{array}
$$

Hint: For proving (1) one must show that $L(r+\emptyset) = L(r)$ holds for each regular expression r, and analogously for the other identities.