Automata and Grammars

SS 2018

Assignment 4: Solutions to Selected Problems

Problem 4.1 [NFAs]

Let $A = (Q, \{a, b\}, \delta, S, F)$ be the NFA that is given by the following table, where $\leftarrow$ indicates an initial state and $\rightarrow$ indicates a final state:

δ	$q_0^{\leftarrow}$	q_1	q_2	q_3	q_4	q_5	$q_{6\rightarrow}$	$q_7^{\leftarrow}$	q_8	q_9	q_{10}	q_{11}	q_{12}	$q_{13\rightarrow}$
a	q_0, q_1	q_2	$-$	q_3	q_5	q_6	q_6	q_7	q_9	q_{10}, q_{11}, q_{12}	q_{10}, q_{11}	q_{12}	$-$	q_{13}
b	q_0	$-$	q_3, q_4	q_3, q_4	$-$	$-$	q_6	q_7, q_8	$-$	$-$	q_{10}	$-$	q_{13}	q_{13}

Determine the sets $\hat{\delta}(P, x)$ for the following sets $P_i \subseteq Q$ and the words $x \in \{aaba, baaa, baba\}$: $P_1 = \{q_0\}$, $P_2 = \{q_7\}$, $P_3 = \{q_2, q_{10}\}$.

Solution. For $P_1 = \{q_0\}$ we obtain the following sets:

q_0	q_0, q_1	q_0, q_1, q_2	q_0, q_3, q_4	q_0, q_1, q_3, q_5
a	a	b	a	

q_0	q_0	q_0, q_1	q_0, q_1, q_2	q_0, q_1, q_2
b	a	a	a	

q_0	q_0	q_0, q_1	q_0	q_0, q_1
b	a	b	a	

For $P_2 = \{q_7\}$ we obtain the following sets:

q_7	q_7	q_7	q_7, q_8	q_7, q_9
a	a	b	a	

q_7	q_7, q_8	q_7, q_9	$q_7, q_{10}, q_{11}, q_{12}$	$q_7, q_{10}, q_{11}, q_{12}$
b	a	a	a	

q_7	q_7, q_8	q_7, q_9	q_7, q_8	q_7, q_9
b	a	b	a	

For $P_3 = \{q_2, q_{10}\}$ we obtain the following sets:

q_2, q_{10}	q_{10}, q_{11}	q_{10}, q_{11}, q_{12}	q_{10}, q_{13}	q_{10}, q_{11}, q_{13}
a	a	b	a	

q_2, q_{10}	q_3, q_4, q_{10}	q_3, q_5, q_{10}, q_{11}	$q_3, q_6, q_{10}, q_{11}, q_{12}$	$q_3, q_6, q_{10}, q_{11}, q_{12}$
b	a	a	a	

q_2, q_{10}	q_3, q_4, q_{10}	q_3, q_5, q_{10}, q_{11}	q_3, q_4, q_{10}	q_3, q_5, q_{10}, q_{11}
b	a	b	a	

$\square$

Problem 4.2 [NFAs]

Construct nondeterministic finite-state automata for the following languages:

$$L_1 = \{ w \in \{a,b\}^* \mid |w| = (3k+2) \text{ for some } k \geq 0 \text{ or } w \text{ ends with } b \} \text{ and}$$
$$L_2 = \{ w \in \{a,b\}^* \mid w \text{ contains the factor } abb \text{ or the factor } bab \}.$$

Solution.

(a) The following NFA $A_1 = (Q_1, \{a,b\}, \delta_1, S_1, F_1)$ accepts the language L_1:

δ_1	$\overrightarrow{q_0}$	q_1	$q_2\leftarrow$	q_3	q_4	$q_5\leftarrow$
a	q_1, q_4	q_2	q_3	q_1	q_4	—
b	q_4, q_5	q_2	q_3	q_1	q_4, q_5	—

(b) The following NFA $A_2 = (Q_2, \{a,b\}, \delta_2, S_2, F_2)$ accepts the language L_2:

δ_2	$\overrightarrow{q_0}$	q_1	q_2	$q_3\leftarrow$	q_4	q_5	$q_6\leftarrow$
a	q_0, q_1	—	—	q_3	q_5	—	q_6
b	q_0, q_4	q_2	q_3	q_3	—	q_6	q_6

□

Problem 4.3 [Power Set Construction]

Use the so-called 'lazy power set construction' to turn the following NFAs into DFAs:

(a) $A_1 = (Q_1, \{a,b\}, \delta_1, S_1, F_1)$ is given by the following table:

δ_1	$\overrightarrow{q_0}$	q_1	q_2	q_3	$q_4\leftarrow$
a	q_0	—	q_3	—	—
b	q_0, q_1	q_2	—	q_4	—

(b) $A_2 = (Q_2, \{a,b\}, \delta_2, S_2, F_2)$ is given by the following table:

δ_2	$\overrightarrow{q_0}$	q_1	q_2	$q_3\leftarrow$	q_4	q_5	$q_6\leftarrow$
a	q_0, q_1	—	—	—	q_5	—	—
b	q_0, q_4	q_2	q_3	—	—	q_6	—

Solution.

(a) The 'lazy' power set construction yields the following DFA B_1 from A_1:

δ_1'	$\overrightarrow{q_0}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_4\}\leftarrow$
a	q_0	q_0	$\{q_0, q_3\}$	q_0	q_0
b	$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_4\}$	$\{q_0, q_1, q_2\}$

(b) The 'lazy' power set construction yields the following DFA B_2 from A_2:

δ_2'	$\overrightarrow{q_0}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_5\}$	$\{q_0, q_4\}$	$\{q_0, q_2, q_4\}$	$\{q_0, q_3, q_4\}\leftarrow$	$\{q_0, q_2, q_4, q_6\}\leftarrow$
a	$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_5\}$	$\{q_0, q_1, q_5\}$	$\{q_0, q_1, q_5\}$	$\{q_0, q_1, q_5\}$
b	$\{q_0, q_4\}$	$\{q_0, q_2, q_4\}$	$\{q_0, q_2, q_4, q_6\}$	$\{q_0, q_4\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_4\}$	$\{q_0, q_3, q_4\}$

□

Problem 4.4 [NFAs and Regular Grammars]
Construct NFAs from the following right regular grammars (see the proof of Theorem 2.16):

(a) $G = (\{S, A, B\}, \{a, b\}, S, P)$, where P contains the following productions:

$$\begin{array}{lllllllll} S & \to & bS, & S & \to & aA, & S & \to & \varepsilon, \\ A & \to & bA, & A & \to & aB, & & & \\ B & \to & bB, & B & \to & aS. & & & \end{array}$$

(b) $G = (\{S, A, B, C\}, \{a, b\}, S, P)$, where P contains the following productions:

$$\begin{array}{lllllllll} S & \to & bA, & & & & & & \\ A & \to & aB, & A & \to & bB, & A & \to & b, \\ B & \to & aC, & B & \to & bC, & B & \to & aB, \\ C & \to & aA, & C & \to & bA, & C & \to & bB. \end{array}$$

Solution.

(b) The NFA $A_2 = (\{S, A, B, C, X\}, \{a, b\}, \delta_2, \{S\}, \{X\})$ looks as follows:

δ_2	S	A	B	C	X
a	—	B	B, C	A	—
b	A	B, X	C	A, B	—

□

Problem 4.5 [ε-NFA]
Let $A = (Q, \{a, b\}, \delta, S, F)$ be the ε-NFA that is described by the following table:

δ	$0^{\leftarrow}$	1	2	3	4	5	6	7	8	9	10	$11_{\rightarrow}$
ε	1	4	—	7	8	—	7	11	—	—	7	—
a	0	2	—	3	—	—	6	7	9	—	—	—
b	0	—	3	3	5	6	6	7	—	10	—	—

Determine the set ε-closure(P_i) for the following subsets P_i of $Q = \{0, 1, 2, \ldots, 11\}$:

(a) $P_1 = \{0\}$, (b) $P_2 = \{3\}$, (c) $P_3 = \{6\}$, (d) $P_4 = \{1, 9\}$.

Hint: See the definition of ε-closure(P) in the proof of Theorem 2.20!

Problem 4.6 [Power Set Construction for ε-NFA]
Determine an equivalent DFA by the 'lazy power set construction' (see the proof of Theorem 2.20) from the ε-NFA A that is given by the following table:

δ_1	$\overleftarrow{q_0}$	q_1	q_2	q_3	q_4	$\overleftarrow{q_5}$	q_6	q_7	q_8	$q_{9\rightarrow}$
ε	—	q_2, q_3	—	—	q_5, q_7	—	—	—	q_5	—
a	q_1	—	q_4	q_8	—	—	—	q_9	—	—
b	—	—	—	—	—	q_6	q_9	—	—	—

Solution. The lazy power set construction yields the following DFA $B_1 = (Q_1', \{a, b\}, \delta_1', \{q_0\}, \{\{q_9\}\})$ from A_1:

δ_1'	$\{q_0, q_5\}$	$\{q_1\}$	$\{q_6\}$	$\{q_9\}$	$\{q_4, q_8\}$	$\emptyset$
a	$\{q_1\}$	$\{q_4, q_8\}$	$\emptyset$	$\emptyset$	$\{q_9\}$	$\emptyset$
b	$\{q_6\}$	$\emptyset$	$\{q_9\}$	$\emptyset$	$\{q_6\}$	$\emptyset$

,

as ε-closure$(\{q_1\}) = \{q_1, q_2, q_3\}$ and ε-closure$(\{q_4, q_8\}) = \{q_4, q_5, q_7, q_8\}$. □