Automata and Grammars

SS 2018

Assignment 5

Solutions are to be presented at the **Seminary** on **Thursday, March 29, 2018**.

Problem 5.1 [2DFA]

Let $A = (Q, \Sigma, \rhd, \lhd, \delta, q_0, F)$ be the 2DFA with initial state p_0 and final state q_4 that is given by the following table:

$\delta:$	p_0	$\overleftarrow{q_0}$	$\overleftarrow{q_1}$	$\overleftarrow{q_2}$	$\overleftarrow{q_3}$	$\overrightarrow{q_0}$	$\overrightarrow{q_1}$	$\overrightarrow{q_2}$	$\overrightarrow{q_3}$	q_4
a	(p_0, R)	$(\overleftarrow{q_1}, L)$	$(\overleftarrow{q_2}, L)$	$(\overleftarrow{q_3}, L)$	$(\overleftarrow{q_2}, L)$	$(\overrightarrow{q_1}, R)$	$(\overrightarrow{q_2}, R)$	$(\overrightarrow{q_3}, R)$	$(\overrightarrow{q_2}, R)$	(q_4, R)
b	(p_0, R)	$(\overleftarrow{q_0}, L)$	$(\overleftarrow{q_0}, L)$	$(\overleftarrow{q_2}, L)$	$(\overleftarrow{q_3}, L)$	$(\overrightarrow{q_0}, R)$	$(\overrightarrow{q_0}, R)$	$(\overrightarrow{q_2}, R)$	$(\overrightarrow{q_3}, R)$	(q_4, R)
c	$(\overleftarrow{q_0}, L)$	$-$	$-$	$-$	$-$	$-$	$-$	(q_4, R)	(q_4, R)	$-$
$\rhd$	(p_0, R)	$(\overrightarrow{q_0}, R)$	$(\overrightarrow{q_1}, R)$	$(\overrightarrow{q_2}, R)$	$(\overrightarrow{q_3}, R)$	$-$	$-$	$-$	$-$	$-$
$\lhd$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$

(a) Describe the step-by-step computation of A on input *abacba*.

(b) Determine all seven crossing sequences of the above computation.

(c) Which language is accepted by A?

Problem 5.2 [Moore Automaton]

Design a Moore automaton with input and output alphabet $\{0, 1\}$ that realizes the function f that is defined as follows:

$$f(a_m a_{m-1} \cdots a_2 a_1) = \begin{cases} 1, & \text{if } \sum_{i=1}^{m} a_i \cdot 2^{i-1} \equiv 0 \mod 4, \\ 0, & \text{otherwise.} \end{cases}$$

Problem 5.3 [Mealey Automaton]

Design a Mealey automaton with input and output alphabet $\{0, 1\}$ that realizes the following function:

- output 1, if the current input symbol is a part of a sequence of 1s, which is directly preceded by the factor 00,
- output 0, otherwise.

Problem 5.4 [Moore and Mealey Automata]

Let $A = (Q, \Sigma, \Delta, \delta, \sigma, q_0)$ be a Moore automaton and let $B = (Q', \Sigma, \Delta, \delta', \sigma', q'_0)$ be a Mealey automaton. For $u \in \Sigma^*$, let $F_A(u) \in \Delta^*$ and $F_B(u) \in \Delta^*$ be the output words that are generated by A and B on input u. The automata A und B are called *equivalent*, if $F_A(u) = \sigma(q_0) \cdot F_B(u)$ for all $u \in \Sigma^*$. Prove the following statements:

(a) For each Moore automaton, there exists an equivalent Mealey automaton.

(b) For each Mealey automaton, there exists an equivalent Moore Automaton.

Problem 5.5 [Finite-State Transducer]

A regular substitution $\varphi : \Sigma^* \to 2^{\Gamma^*}$ maps each letter $a \in \Sigma$ to a regular language $\varphi(a) \in \mathsf{REG}(\Gamma)$ and then $\varphi(a_1 a_2 \cdots a_n) = \varphi(a_1) \cdot \varphi(a_2) \cdots \varphi(a_n)$.

(a) Let $\varphi_1 : \{a,b\}^* \to 2^{\{a,b\}^*}$ be the regular substitution that is given through $a \mapsto \{a\}^*$ and $b \mapsto \{b\}^*$. Construct a finite-state transducer T_1 such that $T_1(w) = \varphi_1(w)$ for each word $w \in \{a,b\}^*$.

(b) Prove that each regular substitution can be realized by a finite-state transducer, that is, if $\varphi : \Sigma^* \to 2^{\Gamma^*}$ is a regular substitution, then there exists a finite-state transducer T such that $T(w) = \varphi(w)$ for each word $w \in \Sigma^*$.