Automata and Grammars

SS 2018

Assignment 4

Solutions are to be presented at the **Seminary** on **Thursday, March 22, 2018**.

Problem 4.1 [NFAs]

Let $A = (Q, \{a, b\}, \delta, S, F)$ be the NFA that is given by the following table, where $\leftarrow$ indicates an initial state and $\rightarrow$ indicates a final state:

δ	$\overleftarrow{q_0}$	q_1	q_2	q_3	q_4	q_5	$q_6{\rightarrow}$	$\overleftarrow{q_7}$	q_8	q_9	q_{10}	q_{11}	q_{12}	$q_{13}{\rightarrow}$
a	q_0, q_1	q_2	$-$	q_3	q_5	q_6	q_6	q_7	q_9	q_{10}, q_{11}, q_{12}	q_{10}, q_{11}	q_{12}	$-$	q_{13}
b	q_0	$-$	q_3, q_4	q_3, q_4	$-$	$-$	q_6	q_7, q_8	$-$	$-$	q_{10}	$-$	q_{13}	q_{13}

Determine the sets $\hat{\delta}(P, x)$ for the following sets $P_i \subseteq Q$ and the words $x \in \{aaba, baaa, baba\}$: $P_1 = \{q_0\}$, $P_2 = \{q_7\}$, $P_3 = \{q_2, q_{10}\}$.

Problem 4.2 [NFAs]

Construct nondeterministic finite-state automata for the following languages:

$$\begin{aligned} L_1 &= \{\, w \in \{a, b\}^* \mid |w| = (3k + 2) \text{ for some } k \geq 0 \text{ or } w \text{ ends with } b \,\} \text{ and} \\ L_2 &= \{\, w \in \{a, b\}^* \mid w \text{ contains the factor } abb \text{ or the factor } bab \,\}. \end{aligned}$$

Problem 4.3 [Power Set Construction]

Use the so-called 'lazy power set construction' to turn the following NFAs into DFAs:

(a) $A_1 = (Q_1, \{a, b\}, \delta_1, S_1, F_1)$ is given by the following table:

δ_1	$\overrightarrow{q_0}$	q_1	q_2	q_3	$q_4{\leftarrow}$
a	q_0	$-$	q_3	$-$	$-$
b	q_0, q_1	q_2	$-$	q_4	$-$

(b) $A_2 = (Q_2, \{a, b\}, \delta_2, S_2, F_2)$ is given by the following table:

δ_2	$\overrightarrow{q_0}$	q_1	q_2	$q_3{\leftarrow}$	q_4	q_5	$q_6{\leftarrow}$
a	q_0, q_1	$-$	$-$	$-$	q_5	$-$	$-$
b	q_0, q_4	q_2	q_3	$-$	$-$	q_6	$-$

Problem 4.4 [NFAs and Regular Grammars]

Construct NFAs from the following right regular grammars (see the proof of Theorem 2.16):

(a) $G = (\{S, A, B\}, \{a, b\}, S, P)$, where P contains the following productions:

$$\begin{array}{lllllllll} S & \to & bS, & S & \to & aA, & S & \to & \varepsilon, \\ A & \to & bA, & A & \to & aB, & & & \\ B & \to & bB, & B & \to & aS. & & & \end{array}$$

(b) $G = (\{S, A, B, C\}, \{a, b\}, S, P)$, where P contains the following productions:

$$\begin{array}{lllllllll} S & \to & bA, & & & & & & \\ A & \to & aB, & A & \to & bB, & A & \to & b, \\ B & \to & aC, & B & \to & bC, & B & \to & aB, \\ C & \to & aA, & C & \to & bA, & C & \to & bB. \end{array}$$

Problem 4.5 [ε-NFA]

Let $A = (Q, \{a, b\}, \delta, S, F)$ be the ε-NFA that is described by the following table:

δ	$0^{\leftarrow}$	1	2	3	4	5	6	7	8	9	10	$11_{\rightarrow}$
ε	1	4	–	7	8	–	7	11	–	–	7	–
a	0	2	–	3	–	–	6	7	9	–	–	–
b	0	–	3	3	5	6	6	7	–	10	–	–

Determine the set ε-closure(P_i) for the following subsets P_i of $Q = \{0, 1, 2, \ldots, 11\}$:

(a) $P_1 = \{0\}$, (b) $P_2 = \{3\}$, (c) $P_3 = \{6\}$, (d) $P_4 = \{1, 9\}$.

Hint: See the definition of ε-closure(P) in the proof of Theorem 2.20!

Problem 4.6 [Power Set Construction for ε-NFA]

Determine an equivalent DFA by the 'lazy power set construction' (see the proof of Theorem 2.20) from the ε-NFA A that is given by the following table:

δ_1	$q_0^{\leftarrow}$	q_1	q_2	q_3	q_4	$q_5^{\leftarrow}$	q_6	q_7	q_8	$q_{9\rightarrow}$
ε	–	q_2, q_3	–	–	q_5, q_7	–	–	–	q_5	–
a	q_1	–	q_4	q_8	–	–	–	q_9	–	–
b	–	–	–	–	–	q_6	q_9	–	–	–