Automata and Grammars

SS 2018

Assignment 3

Solutions are to be presented at the **Seminary** on **Thursday, March 15, 2018**.

Problem 3.1 [Nerode Relation]

Determine all equivalence classes of the Nerode relations $R_L \subseteq \Sigma^* \times \Sigma^*$ for the following languages:

(a) $\Sigma = \{a, b\}$ and $L = \{a, aab, abb\}$,
(b) $\Sigma = \{a, b\}$ and $L = \{ a^m b a^n \mid m, n \geq 1 \}$,
(c) $\Sigma = \{a, b\}$ and $L = \{ a^n b a^n \mid n \geq 0 \}$,

In addition, for those language L, for which R_L has finite index, construct the equivalence class automaton for L.

Problem 3.2 [Myhill Nerode Theorem]

Use the Myhill Nerode Theorem to check which of the following languages are accepted by DFAs:

(a) $L_a = \{ a^n b^n \mid n \geq 1 \}$,
(b) $L_b = \{ ww \mid w \in \{a, b\}^* \}$,
(c) $L_c = \{ a^{2n} \mid n \geq 0 \}$,
(d) $L_d = \{ a^{2^n} \mid n \geq 0 \}$.

Problem 3.3 [DFAs]

Two states p, q of a DFA $A = (Q, \Sigma, \delta, q_0, F)$ are called *equivalent* if, for all $w \in \Sigma^*$, $\delta(p, w) \in F$ iff $\delta(q, w) \in F$. Find all pairs of equivalent states in the following four DFAs:

(a) $A_1 = (\{q_0, q_1, \ldots, q_7\}, \{a, b\}, \delta_1, q_0, \{q_0, q_4, q_6\})$, where δ_1 is described through the following table:

δ_1	q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7
a	q_0	q_1	q_2	q_3	q_6	q_5	q_4	q_7
b	q_5	q_3	q_7	q_2	q_1	q_1	q_2	q_0

(b) $A_2 = (\{A, B, C, D, E, F\}, \{a, b\}, \delta_2, F, \{F\})$, where δ_2 is described by the following table:

δ_2	A	B	C	D	E	F
a	A	B	C	D	E	F
b	F	A	D	B	C	E

(c) $A_3 = (\{q_1, q_2, \ldots, q_9\}, \{a, b\}, \delta_1, q_1, \{q_3, q_5, q_6\})$, where δ_3 is described through the following table:

δ_3	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9
a	q_2	q_2	q_3	q_2	q_6	q_6	q_7	q_2	q_9
b	q_3	q_4	q_5	q_7	q_3	q_6	q_4	q_3	q_4

(d) $A_4 = (\{A, B, C, D, E, F, G, H\}, \{a, b\}, \delta_4, G, \{G\})$, where δ_4 is described by the following table:

δ_4	A	B	C	D	E	F	G	H
a	H	B	E	D	C	F	G	A
b	G	A	D	B	D	E	F	G

Problem 3.4 [Minimal DFA]

Use the algorithm "minimal automaton" to construct the minimal DFAs that are equivalent to the following DFAs:

(a) $B_1 = (\{q_1, q_2, \ldots, q_9\}, \{a, b\}, \delta_1, q_1, \{q_3, q_5, q_6\})$, where δ_1 is described through the following table:

δ_1	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9
a	q_2	q_2	q_3	q_2	q_6	q_6	q_7	q_2	q_9
b	q_3	q_4	q_5	q_7	q_3	q_6	q_4	q_3	q_4

(b) $B_2 = (\{A, B, C, D, E, F, G, H\}, \{a, b\}, \delta_2, G, \{G\})$, where δ_2 is described by the following table:

δ_2	A	B	C	D	E	F	G	H
a	H	B	E	D	C	F	G	A
b	G	A	D	B	D	E	F	G

(c) $B_3 = (\{q_0, q_1, \ldots, q_5\}, \{a, b\}, \delta_3, q_0, \{q_0\})$, where δ_3 is described through the following table:

δ_3	q_0	q_1	q_2	q_3	q_4	q_5
a	q_1	q_3	q_4	q_0	q_2	q_0
b	q_2	q_0	q_5	q_2	q_5	q_3