Automata and Grammars

SS 2018

Assignment 2

Solutions are to be presented at the **Seminary** on **Thursday, March 8, 2018**.

Problem 2.1. [Normal Form for Regular Grammars]

Determine a grammar in *right normal form* that generates the same language as the following grammar $G = (\{S, A, B, C, D\}, \{a, b\}, S, P)$, where P contains the following productions:

$$
\begin{array}{lllllllll}
S & \to & aaA, & S & \to & bbB, & S & \to & C, \\
A & \to & aaA, & A & \to & B, & & & \\
B & \to & bbB, & B & \to & A, & B & \to & C, \\
C & \to & aaA, & C & \to & bbB, & C & \to & D, \\
D & \to & aaA, & D & \to & bbB, & D & \to & \varepsilon.
\end{array}
$$

Hint: Use the construction from the proof of Theorem 2.5.

Problem 2.2 [Closure Properties for Regular Grammars]

Let $G_1 = (N_1, \Sigma, S_1, P_1)$ and $G_2 = (N_2, \Sigma, S_2, P_2)$ be two right regular grammars such that N_1 and N_2 are disjoint.

1. Construct a right regular grammar G from G_1 and G_2 such that $L(G) = L(G_1) \cup L(G_2)$.
2. Construct a right regular grammar G from G_1 and G_2 such that $L(G) = L(G_1) \cap L(G_2)$.
3. Construct a right regular grammar G from G_1 and G_2 such that $L(G) = L(G_1) \cdot L(G_2)$.
4. Construct a right regular grammar G from G_1 such that $L(G) = (L(G_1))^*$.

Hint: You may assume that the grammars G_1 and G_2 are in right normal form.

Problem 2.3 [DFAs]

Construct deterministic finite-state automata for at least three of the following languages:

$$
\begin{array}{lll}
L_1 & = & \{\, w \in \{a,b\}^* \mid |w|_a \text{ is divisible by 2 or 3} \,\}, \\
L_2 & = & \{\, w \in \{a,b\}^* \mid |w|_a \text{ is divisible by 3, but not by 2} \,\}, \\
L_3 & = & \{\, w \in \{a,b\}^* \mid w = uabab \text{ for some word } u \in \{a,b\}^* \,\}, \\
L_4 & = & \{\, w \in \{a,b\}^* \mid w = ababu \text{ for some word } u \in \{a,b\}^* \,\}, \\
L_5 & = & \{\, w \in \{a,b\}^* \mid |w| = (3k+1) \text{ for some } k \geq 0 \text{ or } w \text{ ends with } b \,\}, \\
L_6 & = & \{\, w \in \{a,b\}^* \mid \text{The first and the last letter of } w \text{ are identical} \,\}.
\end{array}
$$

Problem 2.4 [DFAs]

Determine the languages that are accepted by the following deterministic finite-state automata $(\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, F)$, where δ and F are defined as follows:

(a) $F = \{q_1, q_2\}$ and δ:

	0	1
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_0	q_2

,

(b) $F = \{q_2\}$ and δ:

	0	1
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_2

,

(c) $F = \{q_1, q_2\}$ and δ:

	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_0	q_1

.