Automata and Grammars

SS 2018

Assignment 1

Solutions are to be presented at the **Seminary** on **Thursday, March 1, 2018**.

Problem 1.1. [Words]

(a) Let Σ be a finite alphabet. Prove that the operation of concatenation $\cdot : \Sigma^* \times \Sigma^* \to \Sigma^*$ is commutative if and only if Σ has cardinality one.

(b) Show that, for two words $u, v \in \Sigma^*$, $uv = vu$ if and only if there exist a word $x \in \Sigma^*$ and integers $k, l \geq 0$ such that $u = x^k$ and $v = x^l$.

Hint: Use induction on $|u| + |v|$.

Problem 1.2. [Regular Grammars]

Construct regular grammars for at least three of the following languages:

(a) $L_1 = \{ w \in \{a,b\}^* \mid |w|_a \text{ is divisible by 2} \}$,
(b) $L_2 = \{ w \in \{a,b\}^* \mid |w|_a \text{ is divisible by 2 and by 3} \}$,
(c) $L_3 = \{ w \in \{a,b\}^* \mid w = uabab \text{ for some word } u \in \{a,b\}^* \}$,
(d) $L_4 = \{ w \in \{a,b\}^* \mid w \text{ contains the factor } abab \}$,
(e) $L_5 = \{ w \in \{a,b\}^* \mid |w| = (3k+1) \text{ for some } k \geq 0 \text{ or } w \text{ ends with } b \}$,
(f) $L_6 = \{ w \in \{a,b\}^* \mid \text{The first and the last letter of } w \text{ are identical} \}$.

Problem 1.3. [Regular Grammars]

Determine the languages that are generated by the following regular grammars:

(a) $G = (\{S, A, B\}, \{a, b\}, S, P)$, where P contains the following productions:

$$\begin{array}{lllllll} S & \to & aabA, & S & \to & bbaB, \\ A & \to & aabA, & A & \to & aab, \\ B & \to & bbaB, & B & \to & bba. \end{array}$$

(b) $G = (\{S, A, B\}, \{a, b\}, S, P)$, where P contains the following productions:

$$\begin{array}{lllllll} S & \to & abA, & & & \\ A & \to & abA, & A & \to & abB, \\ B & \to & baB, & B & \to & ba. \end{array}$$

(c) $G = (\{S, A, B\}, \{a, b\}, S, P)$, where P contains the following productions:

$$\begin{array}{lllllllll} S & \to & Sb, & S & \to & Aa, & S & \to & \varepsilon, \\ A & \to & Ab, & A & \to & Ba, & & & \\ B & \to & Bb, & B & \to & Sa. & & & \end{array}$$