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II.—ON CONCEPT AND OBJECT

BY GOTTLÖB FREGE

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Translation by P. T. GEACH. (Revised by MAX BLACK.)

192] In a series of articles in this Quarterly on intuition and its psychological elaboration, Benno Kerry has several times referred to my *Grundlagen der Arithmetik* and other works of mine, sometimes agreeing and sometimes disagreeing with me. I cannot but be pleased at this, and I think the best way I can show my appreciation is to take up the discussion of the points he contests. This seems to me all the more necessary, because his opposition is at least partly based on a misunderstanding, which might be shared by others, of what I say about the concept; and because, even apart from this special occasion, the matter is important and difficult enough for a more thorough treatment than seemed to me suitable in my *Grundlagen*.

The word 'concept' is used in various ways; its sense is sometimes psychological, sometimes logical, and sometimes perhaps a confused mixture of both. Since this licence exists, it is natural to restrict it by requiring that when once a usage is adopted it shall be maintained. What I decided was to keep strictly to a purely logical use; the question whether this or that use is more appropriate is one that I should like to leave on one side, as of minor importance. Agreement about the mode of expression will easily be reached when once it is recognized that there is something that deserves a special term.

It seems to me that Kerry's misunderstanding results from his unintentionally confusing his own usage of the word 'concept' with mine. This readily gives rise to contradictions, for which my usage is not to blame.

193] Kerry contests what he calls my definition of 'concept'. I would remark, in the first place, that my explanation is not meant as a proper definition. One cannot require that everything shall be defined, any more than one can require that a chemist shall decompose every substance. What is simple cannot be decomposed, and what is logically simple cannot have a proper definition. Now something logically simple is no more given us

at the outset than most of the chemical elements are ; it is reached only by means of scientific work. If something has been discovered that is simple, or at least must count as simple for the time being, we shall have to coin a term for it, since language will not originally contain an expression that exactly answers. On the introduction of a name for something logically simple, a definition is not possible ; there is nothing for it but to lead the reader or hearer, by means of hints, to understand the words as is intended.

Kerry would like to say that the distinction between concept and object is not absolute. "In a previous passage", he says, "I have myself expressed the opinion that the relation between the content of the concept and the concept-object is, in a certain respect, a peculiar and irreducible one ; but this was in no way bound up with the view that the properties of being a concept and of being an object are mutually exclusive. The latter view no more follows from the former than it would follow, if, *e.g.*, the relation of father and son were one that could not be further reduced, that a man could not be at once a father and a son (though of course not *e.g.* father of the man whose son he was)."

Let us fasten on this simile ! If there were, or had been, beings that were fathers but could not be sons, such beings would obviously be quite different in kind from all men, who are sons. Now it is something like this that happens here. The concept (as I understand the word) is predicative.¹ On the other hand, a name of an object, a proper name, is quite incapable of being used as a grammatical predicate. This admittedly needs elucidation, otherwise it might appear false. Surely one can just as well assert of a thing that it is Alexander the Great, or is the number four, or is the planet Venus, as that it is green or is a mammal ? 194] If anybody thinks this, he is not distinguishing the usages of the word 'is'. In the last two examples it serves as a copula, as a mere verbal sign of predication. (In this sense [the German word *ist*] can sometimes be replaced by the mere personal suffix : *cf. dies Blatt ist grün* and *dies Blatt grünt*.) In such a case we say that something falls under a concept, and the grammatical predicate stands for this concept. In the first three examples, on the other hand, 'is' is used like the 'equals' sign in arithmetic, to express an equation.² In the sentence 'The morning star is

¹ It is, in fact, the reference of a grammatical predicate.

² I use the word 'equal' and the symbol '=' in the sense 'the same as', 'no other than', 'identical with'. *Cf.* E. Schroeder, *Vorlesungen ueber die Algebra der Logik* (Leipzig, 1890), vol. 1, §1. Schroeder must however be criticized for not distinguishing two fundamentally different

Venus', we have two proper names, 'morning star' and 'Venus', for the same object. In the sentence 'the morning star is a planet' we have a proper name, 'the morning star', and a concept-word, 'planet'. So far as language goes, no more has happened than that 'Venus' has been replaced by 'a planet'; but really the relation has become wholly different. An equation is reversible; an object's falling under a concept is an irreversible relation. In the sentence 'the morning star is Venus', 'is' is obviously not the mere copula; its content is an essential part of the predicate, so that the word 'Venus' does not constitute the whole of the predicate.¹ One might say instead: 'the morning star is no other than Venus'; what was previously implicit in the single word 'is' is here set forth in four separate words, and in 'is no other than' the word 'is' now really is the mere copula. What is predicated here is thus not *Venus* but *no other than Venus*. These words stand for a concept; admittedly only one object falls under this, but such a concept must still always be distinguished from the object.² We have here a word 'Venus' that can never be a proper predicate, although it can

195] form part of a predicate. The reference³ of this word is thus something that can never occur as a concept, but only as an object. Kerry too would probably not wish to dispute that there is something of this kind. But this would mean admitting a distinction, which it is very important to recognize, between what can occur only as an object, and everything else. And this distinction would not be effaced even if it were true, as Kerry thinks it is, that there are concepts that can also be objects.

There are, indeed, cases that seem to support his view. I myself have indicated (in *Grundlagen*, §53, *ad fin.*) that a concept may fall under a higher concept—which, however, must not be confused with one concept's being subordinate to another. Kerry does not appeal to this; instead, he gives the following example: "the concept 'horse' is a concept easily attained", and thinks that the concept 'horse' is an object, in fact one of the objects that fall under the concept 'concept easily attained'. Quite so; the three words "the concept 'horse'" do designate an object, but on that very account they do not designate a concept, as I

relations; the relation of an object to a concept it falls under, and the subordination of one concept to another. His remarks on the *Volwurzel* are likewise open to objection. Schroeder's symbol \neq does not simply take the place of the copula.

¹ Cf. my *Grundlagen*, §66, footnote.

² *Ibid.*, §51.

³ Cf. my paper 'On Sense and Reference' (*Ueber Sinn und Bedeutung*), shortly to appear in the *Zeitschrift für Phil. und phil. Kritik*.

am using the word. This is in full accord with the criterion I gave—that the singular definite article always indicates an object, whereas the indefinite article accompanies a concept-word.¹

Kerry holds that no logical rules can be based on linguistic distinctions; but my own way of doing this is something that nobody can avoid who lays down such rules at all; for we cannot come to an understanding with one another apart from language, and so in the end we must always rely on other people's understanding words, inflexions, and sentence-construction in essentially the same way as ourselves. As I said before, I was not trying to give a definition, but only hints; and to this end I appealed to the general feeling for the German language. It is here very much to my advantage that there is such good accord between the linguistic distinction and the real one. As regards the indefinite article there are probably no exceptions to our rule at all for us to remark, apart from obsolete formulas like 'Einedler Rath' ['Councillor']. The matter is not so simple for the definite article, especially in the plural; but 196] then my criterion does not relate to this case. In the singular, so far as I can see, the matter is doubtful only when a singular takes the place of a plural, as in the sentence 'the Turk besieged Vienna', 'the horse is a four-legged animal'. These cases are so easily recognizable as special ones that the value of our rule is hardly impaired by their occurrence. It is clear that in the first sentence 'the Turk' is the proper name of a people. The second sentence is probably best regarded as expressing a universal judgment, say 'all horses are four-legged animals' or 'all properly constituted horses are four-legged animals'; these will be discussed later.² Kerry calls my criterion unsuitable;

¹ *Grundlagen*, §51; §66, footnote; §68, footnote on p. 80.

² Nowadays people seem inclined to exaggerate the scope of the statement that different linguistic expressions are never completely equivalent, that a word can never be exactly translated into another language. One might perhaps go even further, and say that the same word is never taken in quite the same way even by men who share a language. I will not enquire as to the measure of truth in these statements; I would only emphasize that nevertheless different expressions quite often have something in common, which I call the sense, or, in the special case of sentences, the thought. In other words; we must not fail to recognize that the same sense, the same thought, may be variously expressed; thus the difference does not here concern the sense, but only the apprehension, shading, or colouring of the thought, and is irrelevant for logic. It is possible for one sentence to give no more and no less information than another; and, for all the multiplicity of languages, mankind has a common stock of thoughts. If all transformation of the expression were forbidden on the plea that this would alter the content as well, logic would simply be crippled; for the

for surely, he says, in the sentence 'the concept that I am now talking about is an individual concept' the name composed of the first eight words stands for a concept; but he is not taking the word 'concept' in my sense, and it is not in what I have laid down that the contradiction lies. But nobody can require that my mode of expression shall agree with Kerry's.

It must indeed be recognized that here we are confronted by an awkwardness of language, which I admit cannot be avoided, if we say that the concept *horse* is not a concept,¹ whereas, e.g., the [197] city of Berlin is a city and the volcano Vesuvius is a volcano. Language is here in a predicament that justifies the departure from custom. The peculiarity of our case is indicated by Kerry himself, by means of the quotation-marks around 'horse'; I use italics to the same end. There was no reason to mark out the words 'Berlin' and 'Vesuvius' in a similar way. In logical discussions one quite often needs to assert something about a concept, and to express this in the form usual for such assertions, viz., to make what is asserted of the concept into the content of the grammatical predicate. Consequently, one would expect that the reference of the grammatical subject would be the concept; but the concept as such cannot play this part, in view of its predicative nature; it must first be converted into an object,² or, speaking more precisely, represented by an object. We designate this object by prefixing the words 'the concept'; e.g.

'The concept *man* is not empty'.

Here the first three words are to be regarded as a proper name,³ which can no more be used predicatively than 'Berlin' or 'Vesuvius'. When we say 'Jesus falls under the concept *man*', then, setting aside the copula, the predicate is :

'someone falling under the concept *man*'

and this means the same as :

'a man'.

task of logic can hardly be performed without trying to recognize the thought in its manifold guises. Moreover, all definitions would then have to be rejected as false.

¹ A similar thing happens when we say as regards the sentence 'this rose is red': the grammatical predicate 'is red' belongs to the subject 'this rose'. Here the words "The grammatical predicate 'is red'" are not a grammatical predicate but a subject. By the very act of explicitly calling it a predicate, we deprive it of this property.

² Cf. my *Grundlagen*, p. X.

³ I call anything a proper name if it is a sign for an object.

But the phrase

‘ the concept *man* ’

is only part of this predicate.

Somebody might urge, as against the predicative nature of the concept, that nevertheless we speak of a subject-concept. But even in such cases, *e.g.*, in the sentence

‘ all mammals have red blood ’

we cannot fail to recognize the predicative nature¹ of the concept ; for we could say instead :

198] ‘ whatever is a mammal has red blood ’

or : ‘ if anything is a mammal, then it has red blood ’.

When I wrote my *Grundlagen der Arithmetik*, I had not yet made the distinction between sense and reference ;² and so, under the expression ‘ content of a possible judgment ’, I was combining what I now designate by the distinctive words ‘ thought ’ and ‘ truth-value ’. Consequently, I no longer entirely approve of the explanation I then gave (*op. cit.*, p. 77), as regards its wording ; my view is, however, still essentially the same. We may say in brief, taking ‘ subject ’ and ‘ predicate ’ in the linguistic sense : A concept is the reference of a predicate ; An object is something that can never be the whole reference of a predicate, but can be the reference of a subject. It must here be remarked that the words ‘ all ’, ‘ any ’, ‘ no ’, ‘ some ’, are prefixed to concept-words. In universal and particular affirmative and negative sentences, we are expressing relations between concepts ; we use these words to indicate the special kind of relation. They are thus, logically speaking, not to be more closely associated with the concept-words that follow them, but are to be related to the sentence as a whole. It is easy to see this in the case of negation. If in the sentence

‘ all mammals are land-dwellers ’

the phrase ‘ all mammals ’ expressed the logical subject of the predicate *are land-dwellers*, then in order to negate the whole

¹ What I call here the predicative nature of the concept is just a special case of the need of supplementation, the ‘ unsaturatedness ’, that I gave as the essential feature of a function in my work *Funktion und Begriff* (Jena, 1891). It was there scarcely possible to avoid the expression ‘ the function $F(x)$ ’, although there too the difficulty arose that the reference of this expression is not a function.

² Cf. my essay ‘ Sense and Reference ’ in the *Zeitschrift für Phil. und phil. Kritik*.

sentence we should have to negate the predicate : ‘ are not land-dwellers ’. Instead, we must put the ‘ not ’ in front of ‘ all ’ ; from which it follows that ‘ all ’ logically belongs with the predicate. On the other hand, we do negate the sentence ‘ The concept *mammal* is subordinate to the concept *land-dweller* ’ by negating the predicate : ‘ is not subordinate to the concept *land-dweller* ’.

If we keep it in mind that in my way of speaking expressions like ‘ the concept *F* ’ designate not concepts but objects, most of Kerry’s objections

199] already collapse. If he thinks (*cf.* p. 281) that I have identified concept and extension of concept, he is mistaken ; I merely expressed my view that in the expression ‘ the number that applies to the concept *F* is the extension of the concept *equinumerous to the concept F* ’ the words ‘ extension of the concept ’ could be replaced by ‘ concept ’. Notice carefully that here the word ‘ concept ’ is combined with the definite article. Besides this was only a casual remark ; I did not base anything upon it.

Thus Kerry does not succeed in filling the gap between concept and object. Someone might attempt, however, to make use of my own statements in this sense. I have said that to assign a number involves an assertion about a concept ;¹ I speak of properties asserted of a concept, and I allow that a concept may fall under a higher one.² I have called existence a property of a concept. How I mean this to be taken is best made clear by an example. In the sentence ‘ there is at least one square root of 4 ’, we have an assertion, not about (say) the definite number 2, nor about -2 , but about a concept, *square root of 4* ; *viz.*, that it is not empty. But if I express the same thought thus : ‘ The concept *square root of 4* is realized ’, then the first six words form the proper name of an object, and it is about this object that something is asserted. But notice carefully that what is asserted here is not the same thing as was asserted about the concept. This will be surprising only to somebody who fails to see that a thought can be split up in many ways, so that now one thing, now another, appears as subject or predicate. The thought itself does not yet determine what is to be regarded as the subject. If we say ‘ the subject of this judgment ’, we do not designate anything definite unless at the same time we indicate a definite kind of analysis ; as a rule, we do this in connexion with a definite wording. But we must never forget that different sentences may express the

¹ *Grundlagen*, §46.

² *Ibid.*, §53,

same thought. For example, the thought we are considering could also be taken as an assertion about the number 4 :

‘ the number 4 has the property that there is something of which it is the square ’.

Language has means of presenting now one, now another, part 200] of the thought as the subject ; one of the most familiar is the distinction of active and passive forms. It is thus not impossible that one way of analysing a given thought should make it appear as a singular judgment ; another, as a particular judgment ; and a third, as a universal judgment. It need not then surprise us that the same sentence may be conceived as an assertion about a concept and also as an assertion about an object ; only we must observe that what is asserted is different. In the sentence ‘ there is at least one square root of 4 ’ it is impossible to replace the words ‘ square root of 4 ’ by ‘ the concept *square root of 4* ’ ; that is, the assertion that suits the concept does not suit the object. Although our sentence does not present the concept as a subject, it asserts something about it ; it can be regarded as expressing the fact that a concept falls under a higher one.¹ But this does not in any way efface the distinction between object and concept. We see to begin with that in the sentence ‘ there is at least one square root of 4 ’ the predicative nature of the concept is not belied ; we could say ‘ there is something that has the property of giving the result 4 when multiplied by itself ’. Hence what is here asserted about a concept can never be asserted about an object ; for a proper name can never be a predicative expression, though it can be part of one. I do not want to say it is false to assert about an object what is asserted here about a concept ; I want to say it is impossible, senseless, to do so. The sentence ‘ there is Julius Cæsar ’ is neither true nor false but senseless ; the sentence ‘ there is a man whose name is Julius Cæsar ’ has a sense, but here again we have a concept, as the indefinite article shows. We get the same thing in the sentence ‘ there is only one Vienna ’. We must not let ourselves be deceived because language often uses the same word now as a proper name, now as a concept-word ; in our example, the numeral indicates that we have the latter ; ‘ Vienna ’ is here a concept-word, like ‘ metropolis ’. Using it in this sense, we may say : ‘ Trieste is no Vienna ’. If, on the other hand, we substitute ‘ Julius Cæsar ’ 201] for the proper name formed by the first six words of the

¹ In my *Grundlagen* I called such a concept a second-order concept ; in my work *Funktion und Begriff* I called it a second-level concept, as I shall do here.

sentence 'the concept *square root of 4* is realized', we get a sentence that has a sense but is false; for the assertion that something is realized (as the word is being taken here) can be truly made only about a quite special kind of objects, *viz.*, such as can be designated by proper names of the form 'the concept *F*'. Thus the words 'the concept *square root of 4*' have an essentially different behaviour, as regards possible substitutions, from the words 'square root of 4' in our original sentence; that is, the reference of the two phrases is essentially different.¹

What has been shown here in one example holds good generally; the behaviour of the concept is essentially predicative, even where something is being asserted about it; consequently it can be replaced there only by another concept, never by an object. Thus the assertion that is made about a concept does not suit an object. Second-level concepts, which concepts fall under, are essentially different from first-level concepts, which objects fall under. The relation of an object to a first-level concept that it falls under is different from the (admittedly similar) relation of a first-level to a second-level concept. (To do justice at once to the distinction and to the similarity, we might perhaps say: An object falls *under* a first-level concept; a concept falls *within* a second-level concept.) The distinction of concept and object thus still holds, with all its sharpness.²

With this there hangs together what I have said (*Grundlagen*, §53) about my usage of the words 'property' and 'mark'; Kerry's discussion gives me occasion to revert once more to this. The words serve to signify relations, in sentences like ' Φ is a property of Γ ' and ' Φ is a mark of Ω '. In my way of speaking, a thing can be at once a property and a mark, but not of the same thing. I call the concept under which an object falls its properties; thus

'to be Φ is a property of Γ '

¹ Cf. my essay 'Sense and Reference' (cited above).

² [When Russell says that expressions like 'the King of France' are not names but incomplete symbols, he is saying what would be put thus in Frege's terminology: "In 'the King of France is bald', 'the King of France' is not a name of an object; what it stands for is something incomplete, *ungesättigt*—a *second-level* concept, within which the concept *bald* is falsely asserted to fall. The second-level concept in question is the concept: *concept under which somebody falls who is a King of France and apart from whom nobody is a King of France*; no first-level concept falls within this, because nobody is a King of France."

It should, however, be emphasized that Frege himself gives an entirely different account of definite descriptions. Cf. *Ueber Sinn und Bedeutung*, pp. 39-42.—P.T.G.]

is just another way of saying :

‘ Γ falls under the concept of a Φ ’.

If the object Γ has the properties Φ , X , and Ψ , I may combine them into Ω ; so that it is the same thing if I say that Γ has the property Ω , or, that Γ

202] has the properties Φ , X , and Ψ . I then call Φ , X , and Ψ marks of the concept Ω , and, at the same time, properties of Γ . It is clear that the relations of Φ to Γ and to Ω are quite different, and that consequently different terms are required. Γ falls under the concept Φ ; but Ω , which is itself a concept, cannot fall under the first-level concept Φ ; only to a second-level concept could it stand in a similar relation. Ω is, on the other hand, subordinate to Φ .

Let us consider an example, Instead of saying :

‘ 2 is a positive number ’ and
 ‘ 2 is a whole number ’ and
 ‘ 2 is less than 10 ’

we may also say

‘ 2 is a positive whole number less than 10 ’.

Here

to be a positive number,
to be a whole number,
to be less than 10,

appear as properties of the object 2, and also as marks of the concept

positive whole number less than 10.

This is neither positive, nor a whole number, nor less than 10. It is indeed subordinate to the concept *whole number*, but does not fall under it.

Let us now compare with this what Kerry says in his second article (p. 224). “ By the number 4 we understand the result of additively combining 3 and 1. The concept object here occurring is the numerical individual 4 ; a quite definite number in the natural number-series. This object obviously bears just the marks that are named in its concept, and no others besides— provided we refrain, as we surely must, from counting as *propria* of the object its infinitely numerous relations to all other individual numbers ; (‘ the ’ number 4 is likewise the result of additively combining 3 and 1.)”

We see at once that my distinction between property and mark is here quite slurred over. Kerry distinguishes here between the number 4 and 'the' number 4. I must confess that this distinction is incomprehensible to me. The number 4 is to be a concept ; 'the' number 4 is to be a concept-object, and none other than the numerical individual 4. It needs no

203] proof that what we have here is not my distinction between concept and object. It almost looks as though what was floating (though very obscurely) before Kerry's mind were my distinction between the sense and the reference of the words 'the number 4'. But it is only the reference of the words that can be said to be the result of additively combining 3 and 1.

Again, how are we to take the word 'is' in the sentences 'the number 4 is the result of additively combining 3 and 1' and "'the' number 4 is the result of additively combining 3 and 1'" ? Is it a mere copula, or does it help to express a logical equation ? In the first case, 'the' would have to be left out before 'result', and the sentences would go like this :

'The number 4 is a result of additively combining 3 and 1' ;
 "'The' number 4 is a result of additively combining 3 and 1'" .

In that case, the objects that Kerry designates by

'the number 4' and "'the' number 4'"

would both fall under the concept

result of additively combining 3 and 1.

And then the only question would be what difference there was between these objects. (I am here using the words 'object' and 'concept' in my accustomed way.) I should express as follows what Kerry is apparently trying to say :—

'The number 4 has those properties, and those alone, which are marks of the concept : *result of additively combining 3 and 1.*'

I should then express as follows the sense of the first of our two sentences :

'To be a number 4 is the same as being a result of additive combination of 3 and 1' ;

In that case, what I conjectured just now to have been Kerry's intention could also be put thus :—

'The number 4 has those properties, and those alone, which are marks of the concept *a number 4*' .
 (We need not here decide whether this is true.)

204] The inverted commas around the definite article in the words “ ‘ the ’ number 4 ” could in that case be omitted,

But in these attempted interpretations we have assumed that in at least one of the two sentences the definite articles in front of ‘ result ’ and ‘ number 4 ’ were inserted only by an oversight. If we take the words as they stand, we can only regard them as having the sense of a logical equation, like :

‘ The number 4 is none other than the result of additively combining 3 and 1 ’.

The definite article in front of ‘ result ’ is here logically justified only if it is known (i) that there is such a result (ii) that there is not more than one. In that case, the phrase designates an object, and is to be regarded as a proper name. If both of our sentences were to be regarded as logical equations, then, since their right sides are identical, it would follow from them that the number 4 is ‘ the ’ number 4, or, if you prefer, that the number 4 is no other than ‘ the ’ number 4 ; and so Kerry’s distinction would have been proved untenable. However, it is not my present task to point out contradictions in his exposition ; his way of taking the words ‘ object ’ and ‘ concept ’ is not properly my concern here. I am only trying to set my own usage of these words in a clearer light, and incidentally show that in any case it differs from his, whether that is consistent or not.

I do not at all dispute Kerry’s right to use the words ‘ concept ’ and ‘ object ’ in his own way, if only he would respect my equal right, and admit that with my use of terms I have got hold of a distinction of the highest importance. I admit that there is a quite peculiar obstacle in the way of an understanding with my reader. By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought ; I mention an object, when what I intend is a concept. I fully realize that in such cases I was relying upon a reader who would be ready to meet me half-way—who does not begrudge a pinch of salt.

Somebody may think that this is an artificially created difficulty ; that there is no need at all to take account of such an unmanageable thing as what I call a concept ; that one might, like Kerry, regard an object’s falling under a concept as a relation, in which the same thing could occur now as object, now as concept. 205] The words ‘ object ’ and ‘ concept ’ would then serve only to indicate the different positions in the relation. This may be done ; but anybody who thinks the difficulty is avoided this way is very much mistaken ; it is only shifted. For not all the parts of a thought can be complete ; at least one must be ‘ unsaturated ’,

or predicative ; otherwise they would not hold together. For example, the sense of the phrase ' the number 2 ' does not hold together with that of the expression ' the concept *prime number* ' without a link. We apply such a link in the sentence ' the number 2 falls under the concept *prime number* ' ; it is contained in the words ' falls under ' , which need to be completed in two ways—by a subject and an accusative ; and only because their sense is thus ' unsaturated ' are they capable of serving as a link. Only when they have been supplemented in this twofold respect do we get a complete sense, a thought. I say that such words or phrases stand for a relation. We now get the same difficulty for the relation that we were trying to avoid for the concept. For the words ' the relation of an object to the concept it falls under ' designate not a relation but an object ; and the three proper names ' the number 2 ' , ' the concept *prime number* ' , ' the relation of an object to a concept it falls under ' , hold aloof from one another just as much as the first two do by themselves ; however we put them together, we get no sentence. It is thus easy for us to see that the difficulty arising from the ' unsaturatedness ' of one part of the thought can indeed be shifted, but not avoided. ' Complete ' and ' unsaturated ' are of course only figures of speech ; but all that I wish or am able to do here is to give hints.

It may make it easier to come to an understanding if the reader compares my work *Funktion und Begriff*. For over the question what it is that is called a function in Analysis, we come up against the same obstacle ; and on thorough investigation it will be found that the obstacle is essential, and founded on the nature of our language ; that we cannot avoid a certain inappropriateness of linguistic expression ; and that there is nothing for it but to realize this and always take it into account.